Working with fractions.

- <u>What is a fraction</u>**?**

  A fraction is any real number that can be written as the ratio of two integers. This ratio (i.e. quotient) can have any integer as a numerator, but the denominator must not be zero.

  i.e. $\left\{ \text{any real number of the form } \frac{a}{b} \,/\, a \,\&\, b \text{ are integers, and } b \neq 0 \right\}$ the "/" symbol is read **such that.** *a is the numerator, and b is the denominator.*

Examples of fractions: $\frac{2}{5}, \frac{-7}{13}, -\frac{109}{217}, \frac{25}{75}$

- <u>Simplifying.</u>

The fraction $\frac{12}{18}$ can be written in a simpler form if we notice that 12 & 18 have a factor of 6 in common.

$\frac{12}{18} = \frac{6 \cdot 2}{6 \cdot 3} = \frac{2}{3}$, by canceling the 6 from top and bottom. Note we can only cancel factors that are multiplied by each other on the top and bottom.

Note: Terms can not be cancelled $\frac{3+6}{3} \neq 6$ because we can not cancel the 3 since it is a term and not a factor. The answer would be $\frac{3+6}{3} = \frac{9}{3} = 3$ since 3 & 9 share 3 as a common factor.

- <u>Adding and subtracting.</u>

Fractions with **same denominators** can be added or subtracted directly. Construct one big fraction with that same denominator and collect the numerators of each individual fraction as terms and simplify them to a single term if possible via order of operations. Simplify the final fraction if possible.

Fractions with **different denominators** can **<u>only</u>** be added or subtracted if they are changed to equivalent fractions that posses the Least or Lowest Common Denominator-LCD- as their new denominator.

Note: At this point, you may want to review finding the LCD.
(Similar to LCM) Least Common Multiple

The following are three examples on adding and subtracting.

1st Example: $\frac{2}{5}+\frac{8}{5}=\frac{2+8}{5}=\frac{10}{5}=2$

2nd Example: $\frac{2}{5}+\frac{4}{7}=\frac{2}{5}\cdot\frac{7}{7}+\frac{4}{7}\cdot\frac{5}{5}=\frac{2\cdot 7+4\cdot 5}{35}=\frac{34}{35}$, Note the LCD of 5 & & is 35.

3rd example: $\frac{5}{12}-\frac{3}{4}=\frac{5}{12}\cdot\frac{1}{1}-\frac{3}{4}\cdot\frac{3}{3}=\frac{5\cdot 1-3\cdot 3}{12}=\frac{-4}{12}=\frac{-1}{3}$, Note the LCD of 12 & 4 is 12, and

$\frac{-4}{12}=\frac{-1\cdot 4}{3\cdot 4}=\frac{-1}{3}\cdot\frac{4}{4}=\frac{-1}{3}$ where $\frac{4}{4}=1$, here we used canceling.

- <u>Multiplying and dividing.</u>

When multiplying fractions, we simply multiply numerators by each other and denominators by each oth as well. The new fraction will have to be simplified to lowest terms by canceling common factors.

Example: $\frac{5}{12}\cdot\frac{3}{4}=\frac{5\cdot 3}{12\cdot 4}=\frac{15}{48}=\frac{5}{16}$, Note that 15 and 48 have 3 as a common factor. This is usually seen and done in the second step of the process. Note the 3 on top and the 12 on the bottom. They share a factor of 3.

Dividing works the same way once we reciprocate the 2nd fraction in a division problem.

Example: $\frac{5}{12}\div\frac{4}{3}=\frac{5}{12}\cdot\frac{3}{4}=\frac{5\cdot 3}{12\cdot 4}=\frac{15}{48}=\frac{5}{16}$, here the reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$