

Signed Number Arithmetic

Success in performing operations on real numbers, both positive and negative signed numbers, depends on the ability to use the rules for signed number arithmetic and at the same time visualize the operations using the real number line. The rules differ for each operation and must be committed to memory. Adherence to the rules will guarantee correct results.

The Rules

Multiplication

Multiplication of signed numbers can be accomplished for as little as two numbers and up. The numbers are multiplied together, as if they were all positive in their sign, and then the number of negative signs will determine the sign of the final result.

Step 1: Disregard the signs for the moment and multiply the real numbers together and obtain a numeric answer.

Step 2: Count up the number of negative (-) signs in the original problem; If the number of negatives is an even number then your answer is positive. If the number of negatives was an odd number then your answer is negative.

Example $(-2)(1)(-3)(5) = 30$. You see that the problem had 2 (even) negative signs; therefore the final answer is positive (+).

Example $(-1)(4)(-3)\left(-\frac{1}{2}\right) = -6$. You count 3 (odd) negative signs; therefore the final answer is negative (-).

Division

Division is approached the same way as multiplication.

Step 1: Disregard the signs for the moment and do the division of the real numbers and obtain a numeric answer.

Step 2: Count up the number of negative (-) signs in the original problem; If the number of negatives was an even number then your answer is positive. If the number of negatives was an odd number then your answer is negative.

Example $\frac{(-2)(3)(-4)}{(-2)(-3)} = 4$. This fraction had 4 (even) negative signs; therefore the final answer is positive (+).

Example $\left(-\frac{4}{5}\right)\left(\frac{3}{8}\right)\left(-\frac{10}{3}\right)\left(-\frac{48}{24}\right) = -2$. You count 3 (odd) negative signs; therefore the final answer is negative (-).

Addition

Addition of signed (real) numbers comes in two flavors.

Addition of "like" signs

Step 1: Determine if the two numbers that are to be added have the same sign. If so, proceed to step 2.

Step 2: Disregard the signs of the two numbers. Add the two numbers.

Step 3: Use the sign of the original numbers as the sign in your answer.

Example $15 + 11$. The signs of both the 15 and the 11 are positive. More precisely, $+15 + (+11)$. Add the two numbers, $15 + 11 = 26$. Since both of the two numbers were positive (+), the final answer is positive (+).

$$+ 15 + (+11) = 26.$$

Example $-24 + (-13)$. The signs of both 24 and the 13 are negative. Add the two numbers, $24 + 13 = 37$. Since both numbers were negative (-) then the final answer is negative (-). i.e.,

$$-24 + (-13) = -37.$$

Addition of "unlike" signs

Step 1: If the signs of one of the numbers is positive and the other is negative, first determine which is the largest. (i.e., Which has the largest magnitude or absolute value.)

Step 2: Subtract the smaller number from the larger number and obtain a numerical result.

Step 3: Use the sign of the larger number as the sign of the final result.

Example $(-25) + (+7)$. The sign of 25 is negative, the sign of 7 is positive; hence "unlike signs". 25 is larger than 7. First subtracting 7 from 25 yields 18. Since the 25 was largest and negative, then the final answer is negative.

$$(-25) + (+7) = -18.$$

Example $(+12) + (-8)$. The largest number is the 12. Subtracting the smaller from the larger yields 4. $(12 - 8) = 4$. The sign of the largest (12) is positive, therefore, the sign of the final answer is positive.

$$(+12) + (-8) = 4.$$

Subtraction

Subtraction is treated much like addition. There is one small step to do at the beginning of the problem. We will be turning all subtraction problems into addition problems.

Step 1: Change the subtraction operation symbol from $-$ to $+$ and simultaneously change the sign of the subtrahend to the opposite sign. You will be left with an addition problem of either "like" or "unlike" signs.

Step 2: Add the two numbers according to the rules described above.

Example $8 - 15 = (+8) - (+15)$. Change to addition and change the sign of the subtrahend (15) to the opposite sign. $(+8) - (+15) = (+8) + (-15)$ This is an addition problem of "unlike" signs, so subtract the smaller from the larger and take the sign of the larger.

$$(+8) - (+15) = (+8) + (-15) = -7.$$

Example $(-17) - (12) = (-17) - (+12) = (-17) + (-12)$. This is an addition problem of "like" signs, add the two numbers and take their sign.

$$(-17) - (+12) = (-17) + (-12) = -29.$$

Example $(16) - (-15) = (+16) - (-15) = (+16) + (+15) = 31$.

Exponentiation

Raising a positive number to any power results in a positive number.

Example $(5)^3 = 5^3 = 125$

Example $(3)^4 = 3^4 = 81$

The sign of the result of raising a negative number to a power depends on whether the exponent is odd or even. If the exponent is an even number, the sign of the result is positive. If the exponent is an odd number, the sign of the result is negative.

Example $(-3)^4 = 81$. 4 is even which implies positive.

Example $(-5)^3 = -125$. 3 is odd which implies negative.

Lastly, definitely be able to recognize the difference between -5^2 and $(-5)^2$.

Example $-5^2 = -25$

Example $(-5)^2 = +25$

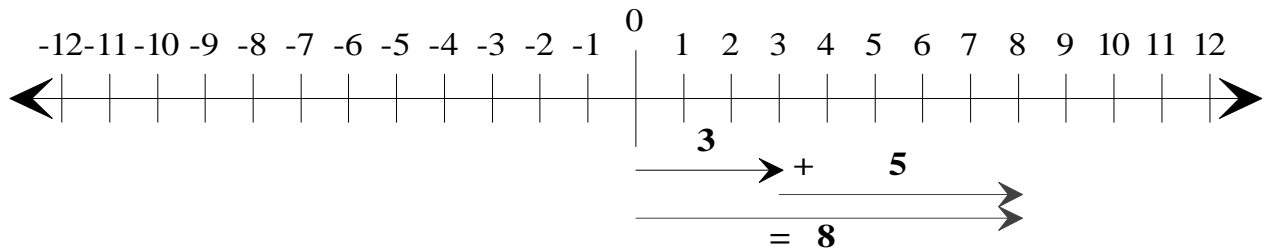
In general, $-x^n \neq (-x)^n$.

The Real Number Line

All of the operations that were outlined above can be performed on the real number line. To see how this works some examples of addition and subtraction will be done. When drawing the arrows for each operation, place the tail of the first arrow at zero and have it point in to the number of the first addend and then draw the arrow tip to tail. The result is read from the origin to the tip of the last arrow.

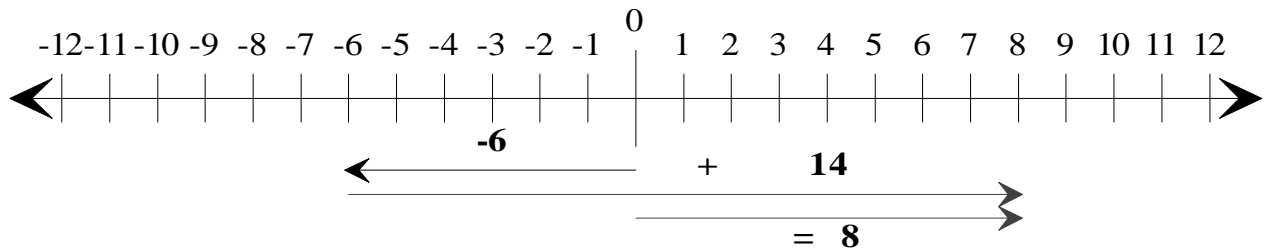
Addition

Example $3 + 5$. Start at the origin and move to the right 3 places to the point 3 on the real number line. Adding positive numbers, like 3 means moving to the right. Next move 5 more places to the right from the point 3 to the point 8. The result is drawn from the tail of the first arrow to the tip of the last arrow.



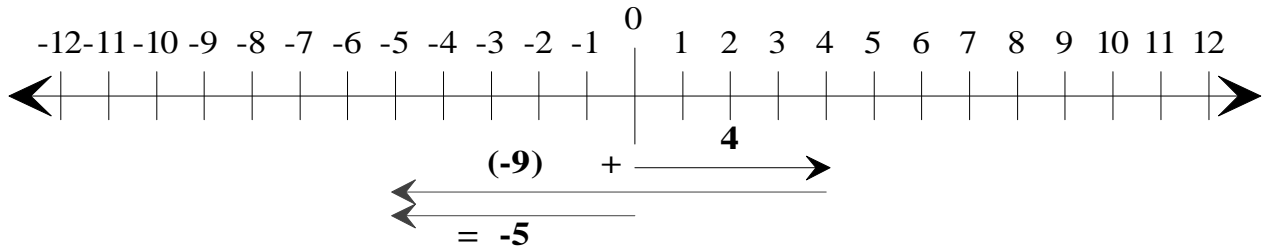
In general, since the signs of the two numbers are the same, if you start out going to the left (for adding two negatives) then you will continue to move to the left. If you start out going to the right (for adding two positives) then you will continue in the same direction to the right.

Example $-6 + 14$. Start at the origin and move to the left 6 places to the point -6 on the real number line. Adding negative numbers, like -6 means moving to the left. Next move 14 places to the right from the point -6 to the point +8. This is the result.



In general, since the two numbers have different signs, (opposite signs), then a switch in direction will occur. If you move right (+) then you will switch and then move left (-). If you move left (-) then you will switch and then move right (+).

Example $4 + (-9)$. Start at the origin and move to the right 4 places to the point 4 on the real number line. Since we are to add a negative number to 4, we next move 9 places to the left from 4 to the point -5. This is the result.

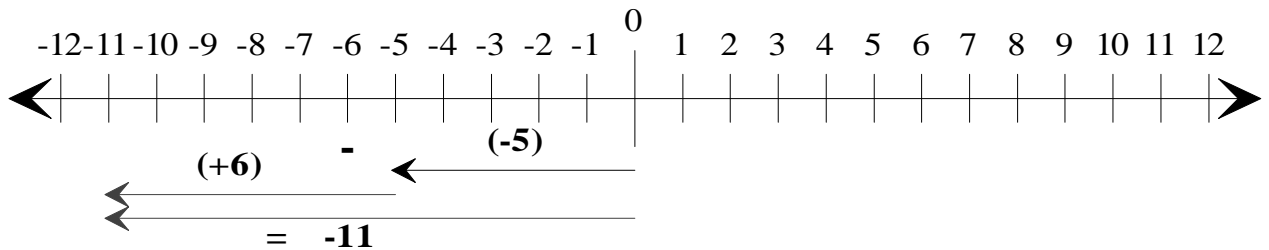


Subtraction

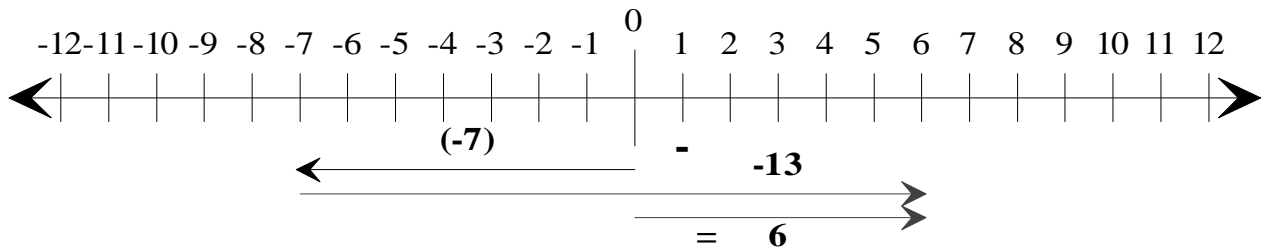
Subtraction is slightly different. Subtracting a positive number means moving to the left.

Subtracting a negative number means moving to the right. (i.e., Subtracting a negative is the same as adding a positive number.)

Example $-5 - 6$. Start at the origin and move to the left 5 places to the point -5. To subtract a positive 6, continue to move to the left another 6 places to the point -11. This is the result.



Example $-7 - (-13)$. Start at the origin and move to the left 7 places to the point -7. Since subtracting a negative number is the same as adding a positive number, move 13 places to the right from the point -7 to the point +6. This is the final result.



The operations of multiplication and division are more difficult to illustrate on the real number line. The rules that were outlined earlier in sections 1 and 2 are the most useful.