EGR 260
Circuit Analysis
File: N260H11BS

Solution to Homework Assignment #11

Problem Assignment: Ch 8 in Electric Circuits, 8th Edition by Nilsson

1) Chapter 8 problems: 3, 4, 6, 30, 35, 38, 40, 44, 51

P 8.3 **[a]** $\alpha = 4000; \qquad \omega_d = 3000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^6 + 16 \times 10^6 = 25 \times 10^6$$

$$\frac{1}{LC} = 25 \times 10^6$$

$$L = \frac{1}{(25 \times 10^6)(50 \times 10^{-9})} = 0.8\,\text{H} = 800\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{10^9}{(8000)(50)} = 2500\,\Omega$$

[c] $V_o = v(0) = 125\,\text{V}$

[d] $I_o = i_\text{L}(0) = -i_\text{R}(0) - i_\text{C}(0)$

$$i_\text{R}(0) = \frac{V_o}{R} = \frac{125}{2.5} \times 10^{-3} = 50\,\text{mA}$$

$$i_\text{C}(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 125\{e^{-4000t}[-3000\sin 3000t - 6000\cos 3000t] -$$

$$4000e^{-4000t}[\cos 3000t - 2\sin 3000t]$$

$$\frac{dv}{dt}(0) = 125\{1(-6000) - 4000\} = -125 \times 10^4$$

$$C\frac{dv}{dt}(0) = -125 \times 10^4(50 \times 10^{-9}) = -6250 \times 10^{-5} = -62.5\,\text{mA}$$

$$\therefore \quad I_o = -50 + 62.5 = 12.5\,\text{mA}$$

[e] $\dfrac{dv}{dt} = 125e^{-4000t}[5000\sin 3000t - 10{,}000\cos 3000t]$

$$= 625 \times 10^3 e^{-4000t}[\sin 3000t - 2\cos 3000t]$$

$$C\frac{dv}{dt} = 31{,}250 \times 10^{-6} e^{-4000t}(\sin 3000t - 2\cos 3000t)$$

$$i_\text{C}(t) = 31.25e^{-4000t}(\sin 3000t - 2\cos 3000t)\,\text{mA}$$

$$i_\text{R}(t) = 50e^{-4000t}(\cos 3000t - 2\sin 3000t)\,\text{mA}$$

$$i_\text{L}(t) = -i_\text{R}(t) - i_\text{C}(t)$$

$$= e^{-4000t}(12.5\cos 3000t + 68.75\sin 3000t)\,\text{mA}, \quad t \geq 0$$

CHECK:

$$\frac{di_\text{L}}{dt} = \{-4000e^{-4000t}[12.5\cos 3000t + 68.75\sin 3000t]$$

$$+e^{-4000t}[-37.5 \times 10^3 \sin 3000t$$

$$+206.25 \times 10^3 \cos 3000t] \times 10^{-3}$$

$$= e^{-4000t}[156.25\cos 3000t - 312.5\sin 3000t]$$

$$L\frac{di_\text{L}}{dt} = e^{-4000t}[125\cos 3000t - 250\sin 3000t]$$

$$= 125e^{-4000t}[\cos 3000t - 2\sin 3000t]\,\text{V}$$

P 8.4 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = (4000)^2$

$$\therefore \quad C = \frac{1}{(16\times 10^6)(5)} = 12.5\,\text{nF}$$

$$\frac{1}{2RC} = 4000$$

$$\therefore \quad R = \frac{10^9}{(8000)(12.5)} = 10\,\text{k}\Omega$$

$$v(0) = D_2 = 25\,\text{V}$$

$$i_\text{R}(0) = \frac{25}{10} = 2.5\text{mA}$$

$$i_\text{C}(0) = -2.5 - 5 = -7.5\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 4000D_2 = \frac{-7.5\times 10^{-3}}{12.5\times 10^{-9}} = -6\times 10^5$$

$$\therefore \quad D_1 = -6\times 10^5 + 4000(25) = -5\times 10^5\ \text{V/s}$$

[b] $v = -5\times 10^5 te^{-4000t} + 25e^{-4000t}$

$$\frac{dv}{dt} = [20\times 10^8 t - 6\times 10^5]e^{-4000t}$$

$$i_\text{C} = C\frac{dv}{dt} = 12.5\times 10^{-9}[20\times 10^8 t - 6\times 10^5]e^{-4000t}$$

$$= (25{,}000t - 7.5)e^{-4000t}\,\text{mA}, \qquad t > 0$$

P 8.6 **[a]** $i_\text{R}(0) = \dfrac{90}{2000} = 45\text{mA}$

$$i_\text{L}(0) = -30\text{mA}$$

$$i_\text{C}(0) = -i_\text{L}(0) - i_\text{R}(0) = 30 - 45 = -15\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(250)(10)} = 4\times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{6.25\times 10^8 - 10^8(4)} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000\ \text{rad/s}; \qquad s_2 = -40{,}000\ \text{rad/s}$$

$$v = A_1 e^{-10{,}000t} + A_2 e^{-40{,}000t}$$

$$v(0) = A_1 + A_2 = 90$$

$$\frac{dv}{dt}(0) = -10^4 A_1 - 4A_2\times 10^4 = \frac{-15\times 10^{-3}}{10\times 10^{-9}} = -1.5\times 10^6\text{V/s}$$

$$-A_1 - 4A_2 = -150$$

$$\therefore \quad -3A_2 = -60; \qquad A_2 = 20; \qquad A_1 = 70$$

$$v = 70e^{-10{,}000t} + 20e^{-40{,}000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_\text{C} = C\dfrac{dv}{dt}$

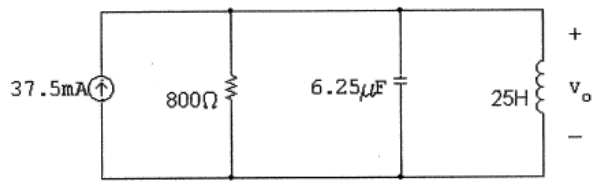
$$= 10\times 10^{-9}[-70\times 10^4 e^{-10{,}000t} - 80\times 10^4 e^{-40{,}000t}]$$

$$= -7e^{-10{,}000t} - 8e^{-40{,}000t}\,\text{mA}$$

$$i_\text{R} = 35e^{-10{,}000t} + 10e^{-40{,}000t}\,\text{mA}$$

$$i_\text{L} = -i_\text{C} - i_\text{R} = -28e^{-10{,}000t} - 2e^{-40{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.30 For $t > 0$



$$\alpha = \frac{1}{2RC} = 100; \qquad \frac{1}{LC} = 6400$$

$$s_{1,2} = -100 \pm 60$$

$$s_1 = -40\,\text{rad/s}; \qquad s_2 = -160\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 37.5\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 6000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -40A_1' - 160A_2'$$

$$-40A_1' - 160A_2' = 6000$$

$$A_1' + 4A_2' = -150$$

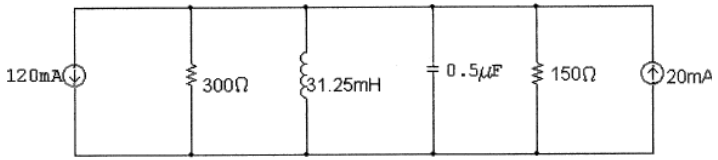
$$A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 50\,\text{V}; \qquad A_2' = -50\,\text{V}$$

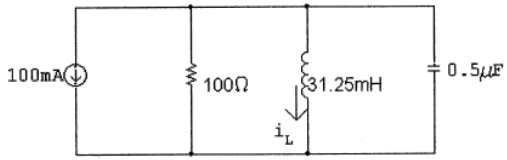
$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 8.35 $t < 0$: $i_L = 3/150 = 20\,\text{mA}$

$t > 0$:



$300 \| 150 = 100\,\Omega$



$$i_L(0) = 20\,\text{mA}, \qquad i_L(\infty) = -100\,\text{mA}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(31.25)(0.5)} = 64 \times 10^6; \qquad \omega_o = 8000\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(200)(0.5)} = 10^4; \qquad \alpha^2 = 100 \times 10^6$$

$$\alpha^2 - \omega_o^2 = (100 - 64)10^6 = 36 \times 10^6$$

$$s_{1,2} = -10{,}000 \pm 6000$$

$$s_1 = -4000\ \text{rad/s}; \qquad s_2 = -16{,}000\ \text{rad/s}$$

$$i_L = I_f + A_1' e^{-4000t} + A_2' e^{-16{,}000t}$$

$$i_L(\infty) = I_f = -100\text{mA}$$

$$i_L(0) = A_1' + A_2' + I_f = 20\,\text{mA}$$

P 8.38 $\alpha = 800\,\text{rad/s}; \qquad \omega_d = 600\,\text{rad/s}$

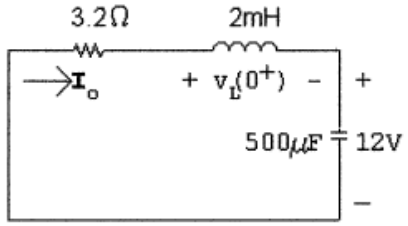
$$\omega_o^2 - \alpha^2 = 36 \times 10^4; \qquad \omega_o^2 = 100 \times 10^4; \quad w_o = 1000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 800; \qquad R = 1600L$$

$$\frac{1}{LC} = 100 \times 10^4; \qquad L = \frac{10^6}{(100 \times 10^4)(500)} = 2\,\text{mH}$$

$$\therefore \quad R = 3.2\,\Omega$$

$$i(0^+) = B_1 = 0\,\text{A}; \qquad \text{at } t = 0^+$$



$$12 + 0 + v_L(0^+) = 0; \qquad v_L(0^+) = -12\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-12}{0.002} = -6000\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 600B_2 - 800B_1 = -6000$$

$$\therefore \quad 600B_2 = 800B_1 - 6000; \qquad \therefore \quad B_2 = -10\,\text{A}$$

$$\therefore \quad i = -10e^{-800t}\sin 600t\,\text{A}, \quad t \geq 0$$

P 8.40 [a] $t < 0$:

$$i_o = \frac{120}{8000} = 15\,\text{mA}; \qquad v_o = (5000)(0.015) = 75\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{5000}{2(1)} = 2500\ \text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1)(250)} = 4 \times 10^6 = 400 \times 10^4$$

$$\alpha^2 - \omega_o^2 = 625 \times 10^4 - 400 \times 10^4 = 225 \times 10^4$$

$$\therefore \quad s_{1,2} = -2500 \pm 1500$$

$$s_1 = -1000\ \text{rad/s} \qquad s_2 = -4000\ \text{rad/s}$$

$$\therefore \quad i_o(t) = A_1e^{-1000t} + A_2e^{-4000t}$$

$$i_o(0) = A_1 + A_2 = 15 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = -1000A_1 - 4000A_2 = 0$$

Solving, $\qquad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$

$$i_o(t) = 20e^{-1000t} - 5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$$

[b] $v_o(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$v_o(0) = A_1 + A_2 = 75$$

$$\frac{dv_o}{dt}(0) = -1000A_1 - 4000A_2 = \frac{-15 \times 10^{-3}}{250 \times 10^{-9}}$$

Solving, $\qquad A_1 = 80\,\text{V}; \qquad A_2 = -5\,\text{V}$

$$v_o(t) = 80e^{-1000t} - 5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

Check:

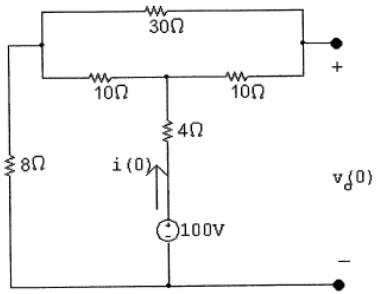
$$5000i_o + 1\frac{di_o}{dt} = v_o$$

$$5000i_o = 100e^{-1000t} - 25e^{-4000t}$$

$$\frac{di_o}{dt} = -20e^{-1000t} + 20e^{-4000t}$$

$$\therefore \quad 5000i_o + \frac{di_o}{dt} = 80e^{-1000t} - 5e^{-4000t}\,\text{V} \qquad \text{(checks)}$$

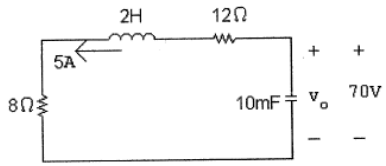
P 8.44 $t < 0$:



$$i(0) = \frac{100}{4+8+8} = \frac{100}{20} = 5\,\text{A}$$

$$v_o(0) = 100 - 5(4) - 10(5)\left(\frac{10}{50}\right) = 70\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{4} = 5, \qquad \alpha^2 = 25$$

$$\omega_o^2 = \frac{1}{LC} = \frac{100}{2} = 50$$

$\omega_o^2 > \alpha^2$ underdamped

$$v_o = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t; \qquad \omega_d = \sqrt{50-25} = 5$$

$$v_o = B_1 e^{-5t}\cos 5t + B_2 e^{-5t}\sin 5t$$

$$v_o(0) = B_1 = 70\,\text{V}$$

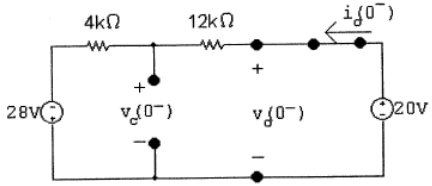
$$C\frac{dv_o}{dt}(0) = -5, \qquad \frac{dv_o}{dt} = \frac{-5}{10}\times 10^3 = -500\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -5B_1 + 5B_2 = -500$$

$$5B_2 = -500 + 5B_1 = -500 + 350; \qquad B_2 = -150/5 = -30\,\text{V}$$

$$\therefore \quad v_o = 70e^{-5t}\cos 5t - 30e^{-5t}\sin 5t\,\text{V}, \qquad t \geq 0$$

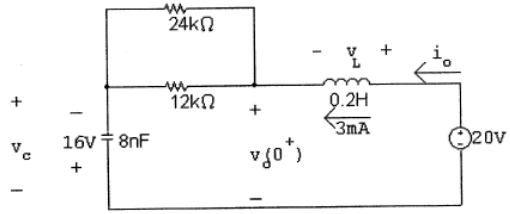
P 8.51 [a] $t < 0$:



$$i_o(0^-) = \frac{48}{16{,}000} = 3\,\text{mA}$$

$$v_C(0^-) = 20 - (12{,}000)(0.003) = -16\,\text{V}$$

$t = 0^+$:



$$12\,\text{k}\Omega \| 24\,\text{k}\Omega = 8\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = (0.003)(8000) - 16 = 24 - 16 = 8\,\text{V}$$

and $v_L(0^+) = 20 - 8 = 12\,\text{V}$

[b] $v_o(t) = 8000 i_o + v_C$

$$\frac{dv_o}{dt}(t) = 8000\frac{di_o}{dt} + \frac{dv_C}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 8000\frac{di_o}{dt}(0^+) + \frac{dv_C}{dt}(0^+)$$

$$v_L(0^+) = L\frac{di_o}{dt}(0^+)$$

$$\frac{di_o}{dt}(0^+) = \frac{v_L(0^+)}{L} = \frac{12}{0.2} = 60\ \text{A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{3\times 10^{-3}}{8\times 10^{-9}} = 375{,}000$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 8000(60) + 375{,}000 = 855{,}000\ \text{V/s}$$

8.51 (continued)

[c] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{1.6} = 625 \times 10^6; \qquad \omega_o = 25{,}000 \text{ rad/s}$

$\alpha = \dfrac{R}{2L} = \dfrac{8000}{0.4} = 20{,}000 \text{ rad/s}; \qquad \alpha^2 = 400 \times 10^6$

$\alpha^2 < \omega_o^2$ underdamped

$s_{1,2} = -20{,}000 \pm j15{,}000 \text{ rad/s}$

$v_o(t) = V_f + B_1' e^{-20{,}000t} \cos 15{,}000t + B_2' e^{-20{,}000t} \sin 15{,}000t$

$V_f = v_o(\infty) = 20 \text{ V}$

$8 = 20 + B_1'; \qquad B_1' = -12 \text{ V}$

$-20{,}000B_1' + 15{,}000B_2' = 855{,}000$

Solving, $\qquad B_2' = 41 \text{ V}$

$\therefore \quad v_o(t) = 20 - 12e^{-20{,}000t} \cos 15{,}000t + 41e^{-20{,}000t} \sin 15{,}000t \text{ V}, \qquad t \geq 0^+$

2) Graph the following responses using MathCAD, MatLab, or Excel and turn in printouts of each graph. Include a title that identifies the problem number and indicates whether the graph is overdamped, underdamped, or critically damped. Graph each response from 0 to 5Tau and use good rules for graphing, including a sufficient number of points so that each graph is smooth.

A) $i(t) = 20e^{-100t} + 30e^{-500t}$ A, $t \geq 0$

B) $i(t) = 20e^{-5,000t} - 30e^{-20,000t}$ A, $t \geq 0$

C) $i(t) = (20,000t+2)e^{-400t}$ A, $t \geq 0$

D) $i(t) = e^{-10,000t}[30\cos(4000\pi t)+40\cos(4000\pi t)]$ A, $t \geq 0$

E) $i(t) = e^{-2,500t}[30\cos(4000\pi t)+40\cos(4000\pi t)]$ A, $t \geq 0$

F) $i(t) = e^{-500t}[30\cos(4000\pi t)+40\cos(4000\pi t)]$ A, $t \geq 0$

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% EGR 260 - Circuit Analysis
% Solution to Homework #8
% Part 2A _____________________________
t=0.0:0.0005:0.05;   % 101 points
iA = 20*exp(-100*t)+20*exp(-500*t);
figure (1)
plot(t,iA)
title('Part 2A - Overdamped')
xlabel('time, t (s)')
ylabel('current, iA (A)')
% Part 2B _____________________________
t=0.0:0.000025:0.0025;  % 101 points
iB = 20*exp(-5000*t)-20*exp(-20000*t);
figure (2)
plot(t,iB)
title('Part 2B - Overdamped')
xlabel('time, t (s)')
ylabel('current, iB (A)')
% Part 2C _____________________________
t=0.0:0.000125:0.0125;  % 101 points
iC = (20000*t+2).*exp(-400*t);
figure (3)
plot(t,iC)
title('Part 2C - Critically Damped')
xlabel('time, t (s)')
ylabel('current, iC (A)')
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% Part 2D _____________________________
t=0.0:0.000005:0.0005;   % 101 points
iD = exp(-10000*t).*(30*cos(4000*pi*t) + 40*sin(4000*pi*t));
figure (4)
plot(t,iD)
title('Part 2D - Underdamped')
xlabel('time, t (s)')
ylabel('current, iD (A)')
% Part 2E _____________________________
t=0.0:0.00002:0.002;   % 101 points
iE = exp(-2500*t).*(30*cos(4000*pi*t) + 40*sin(4000*pi*t));
figure (5)
plot(t,iE)
title('Part 2E - Underdamped')
xlabel('time, t (s)')
ylabel('current, iE (A)')
% Part 2F _____________________________
t=0.0:0.00001:0.01;   % 101 points
iF = exp(-500*t).*(30*cos(4000*pi*t) + 40*sin(4000*pi*t));
figure (6)
plot(t,iF)
title('Part 2F - Underdamped')
xlabel('time, t (s)')
ylabel('current, iF (A)')
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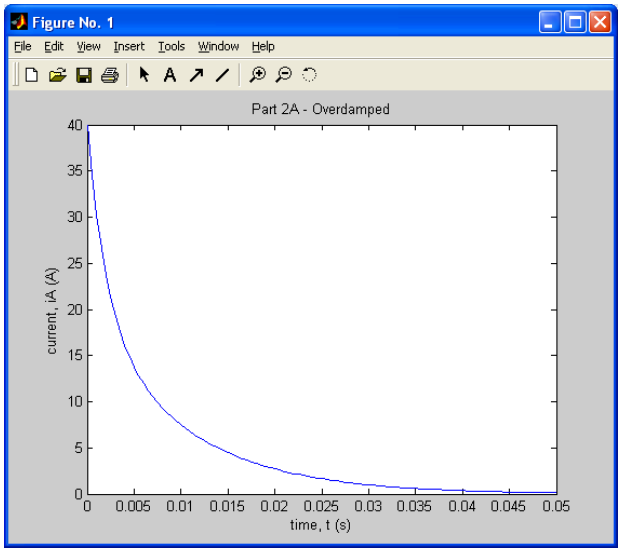
Figure No. 1
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Part 2A - Overdamped
current, iA (A)
time, t (s)

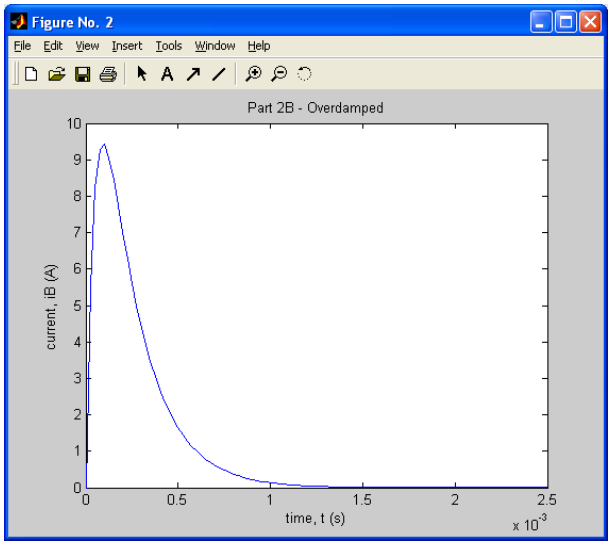
Figure No. 2
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Part 2B - Overdamped
current, iB (A)
time, t (s)
x 10^-3

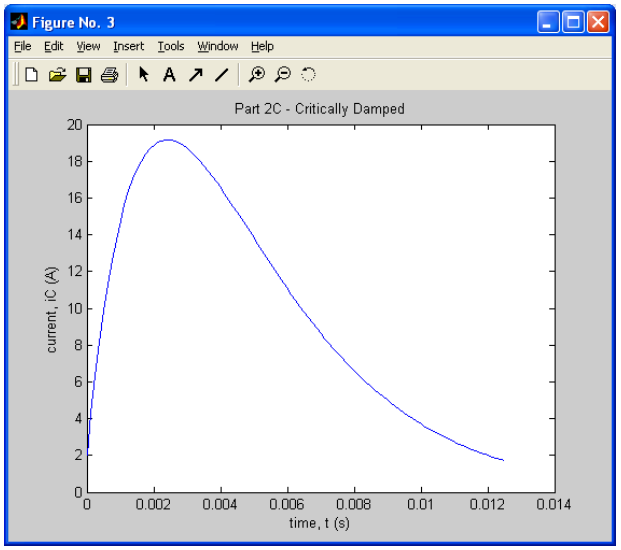
Figure No. 3
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Part 2C - Critically Damped
current, iC (A)
time, t (s)

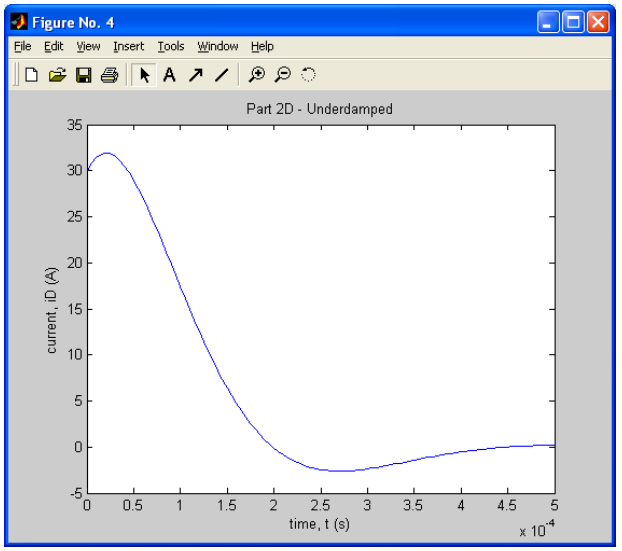
Figure No. 4
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Part 2D - Underdamped
current, iD (A)
time, t (s)
x 10^-4

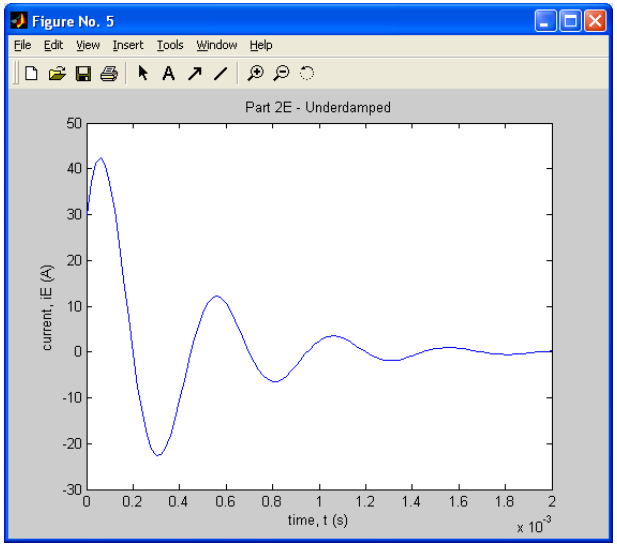
Figure No. 5
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Part 2E - Underdamped
current, iE (A)
time, t (s)
x 10^-3

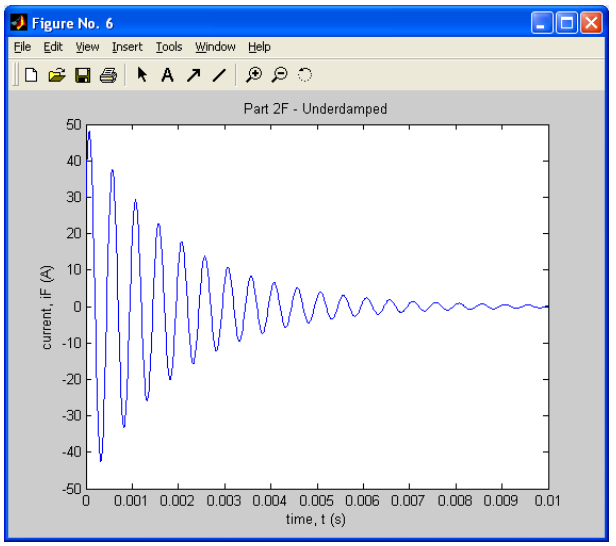
Figure No. 6
File Edit View Insert Tools Window Help
Part 2F - Underdamped
current, iF (A)
time, t (s)