EGR 260
Circuit Analysis
File: N260H1BS

Solution to Homework Assignment #1

Problem Assignment:

1) Chapter 1 # 9, 11, 12, 15, 16, 17, 19, 20, 22, 28, 30
2) Using the power rating values from Table 1 (or from the Lecture #2 notes), calculate the following if the cost of electricity is $0.08/kW·h:
   A) The cost to run an air conditioner 12 h/day for 30 days
   B) The cost to watch a television for 4 h/day for 1 year
   C) The cost to dry your hair with a blow dryer (10 minutes)

Solutions:

P 1.9 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 24\cos 4000t$$

Therefore, $dq = 24\cos 4000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 24\int_0^t \cos 4000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 24\frac{\sin 4000y}{4000}\Big|_0^t = \frac{24}{4000}\sin 4000t - \frac{24}{4000}\sin 4000(0) = \frac{24}{4000}\sin 4000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6\times 10^{-3}\sin 4000t\,\text{C} = 6\sin 4000t\,\text{mC}$

P 1.11 $p = (9)(100\times 10^{-3}) = 0.9$ W; $\quad 5\text{ hr}\cdot\frac{3600\text{ s}}{1\text{ hr}} = 18{,}000$ s

$$w(t) = \int_0^t p\,dt \qquad w(18{,}000) = \int_0^{18{,}000} 0.9\,dt = 0.9(18{,}000) = 16.2\text{ kJ}$$

P 1.12 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (120)(5) = 600$ W   600 W from A to B

[b] $p = (250)(-8) = -2000$ W   2000 W from B to A

[c] $p = (-150)(16) = -2400$ W   2400 W from B to A

[d] $p = (-480)(-10) = 4800$ W   4800 W from A to B

P 1.15 [a] In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,
$p = vi = (30)(12) = 360$ W.
Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

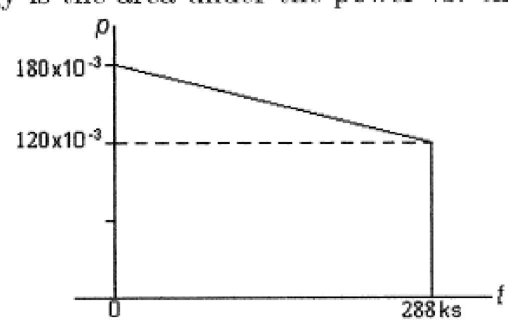
[b] $w(t) = \int_0^t p\,dx;$ 1 min = 60 s

$$w(60) = \int_0^{60} 360\,dx$$

$$w = 360(60 - 0) = 360(60) = 21{,}600 \text{ J} = 21.6 \text{ kJ}$$

P 1.16 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 80 hr = 288,000 s = 288 ks

$$p(0) = (9)(20 \times 10^{-3}) = 180 \times 10^{-3} \text{ W}$$

$$p(288 \text{ ks}) = (6)(20 \times 10^{-3}) = 120 \times 10^{-3} \text{ W}$$

$$w = (120 \times 10^{-3})(288 \times 10^{3}) + \frac{1}{2}(180 \times 10^{-3} - 120 \times 10^{-3})(288 \times 10^{3}) = 43.2 \text{ kJ}$$

P 1.17 [a] $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$

$p(1 \text{ ms}) = 3.1 \text{ mW}$

[b]

$$\begin{aligned} w(t) &= \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + \\ &\qquad 50e^{-2000x} - 10e^{-3000x})dx \\ &= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - \\ &\qquad 25e^{-2000t} + 3.33e^{-3000t}\mu\text{J} \\ w(1 \text{ ms}) &= 1.24\mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 21.67\mu\text{J}$

P 1.19 [a] $0 \text{ s} \le t < 4 \text{ s}$:

$v = 2.5t$ V; $i = 1\,\mu$A; $p = 2.5t\,\mu$W

$4 \text{ s} < t \le 8 \text{ s}$:

$v = 10$ V; $i = 0$ A; $p = 0$ W

$8 \text{ s} \le t < 16 \text{ s}$:

$v = -2.5t + 30$ V; $i = -1\,\mu$A; $p = 2.5t - 30\,\mu$W

$16 \text{ s} < t \le 20 \text{ s}$:

$v = -10$ V; $i = 0$ A; $p = 0$ W

$20 \text{ s} \le t < 36 \text{ s}$:

$v = t - 30$ V; $i = 0.4\,\mu$A; $p = 0.4t - 12\,\mu$W

$36 \text{ s} < t \le 46 \text{ s}$:

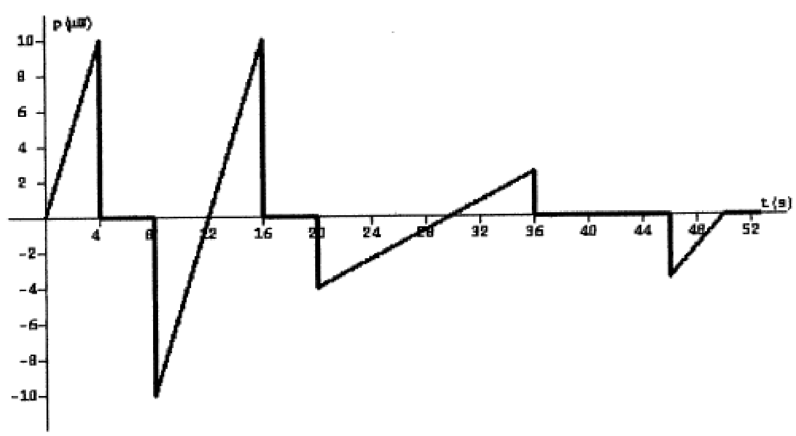
$v = 6$ V; $i = 0$ A; $p = 0$ W

$46 \text{ s} \le t < 50 \text{ s}$:

$v = -1.5t + 75$ V; $i = -0.6\,\mu$A; $p = 0.9t - 45\,\mu$W

$t > 50$ s:

$v = 0$ V; $i = 0$ A; $p = 0$ W



[b] Calculate the area under the curve from zero up to the desired time:

$$\begin{aligned}
w(4) &= \tfrac{1}{2}(4)(10) = 20\,\mu\text{J} \\
w(12) &= w(4) - \tfrac{1}{2}(4)(10) = 0 \text{ J} \\
w(36) &= w(12) + \tfrac{1}{2}(4)(10) - \tfrac{1}{2}(10)(4) + \tfrac{1}{2}(6)(2.4) = 7.2\,\mu\text{J} \\
w(50) &= w(36) - \tfrac{1}{2}(4)(3.6) = 0 \text{ J}
\end{aligned}$$

P 1.20 [a] $p = vi = (0.05e^{-1000t})(75 - 75e^{-1000t}) = (3.75e^{-1000t} - 3.75e^{-2000t})$ W

$\frac{dp}{dt} = -3750e^{-1000t} + 7500e^{-2000t} = 0$ so $2e^{-2000t} = e^{-1000t}$

$2 = e^{1000t}$ so $\ln 2 = 1000t$ thus p is maximum at $t = 693.15\,\mu$s

$p_{\text{max}} = p(693.15\,\mu\text{s}) = 937.5$ mW

[b] $w = \int_0^\infty [3.75e^{-1000t} - 3.75e^{-2000t}]\,dt = \left[\frac{3.75}{-1000}e^{-1000t} - \frac{3.75}{-2000}e^{-2000t}\Big|_0^\infty\right]$

$= \frac{3.75}{1000} - \frac{3.75}{2000} = 1.875$ mJ

P 1.22 [a]

$$q = \text{area under } i \text{ vs. } t \text{ plot}$$
$$= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right] \times 10^3$$
$$= [10 + 40 + 16 + 48 + 9]10^3 = 123{,}000 \text{ C}$$

[b]

$$w = \int p dt = \int vi\, dt$$
$$v = 0.2 \times 10^{-3}t + 9 \qquad 0 \le t \le 15 \text{ ks}$$

$0 \le t \le 4000s$

$$i = 15 - 1.25 \times 10^{-3}t$$
$$p = 135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2$$
$$w_1 = \int_0^{4000} (135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2)\, dt$$
$$= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ}$$

$4000 \le t \le 12{,}000$

$$i = 12 - 0.5 \times 10^{-3}t$$
$$p = 108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2$$
$$w_2 = \int_{4000}^{12{,}000} (108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2)\, dt$$
$$= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ}$$

$12{,}000 \le t \le 15{,}000$

$$i = 30 - 2 \times 10^{-3}t$$
$$p = 270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2$$
$$w_3 = \int_{12{,}000}^{15{,}000} (270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2)\, dt$$
$$= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ}$$
$$w_T = w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ}$$

2A) Since power is constant, W = P*t = (860W)(12 h/day)(30 days) = 309.6 kW·h
Cost = (309.6 kW·h)($0.08/kW·h) = $24.77

$2B) Since power is constant, W = P*t = (150W)(4 h/day)(365 days) = 219 kW·h
Cost = (219 kW·h)($0.08/kW·h) = $17.52

2C) Since power is constant, W = P*t = (1300W)(10 min)(1h/60 min) = 0.2667 kW·h
Cost = (0.26667 kW·h)($0.08/kW·h) = $0.017

P 1.28

$$
\begin{aligned}
p_{\rm a} &= -v_{\rm a}i_{\rm a} = -(36)(250\times10^{-6}) = -9\ \text{mW}\\
p_{\rm b} &= v_{\rm b}i_{\rm b} = (44)(-250\times10^{-6}) = -11\ \text{mW}\\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (28)(-250\times10^{-6}) = -7\ \text{mW}\\
p_{\rm d} &= v_{\rm d}i_{\rm d} = (-108)(100\times10^{-6}) = -10.8\ \text{mW}\\
p_{\rm e} &= v_{\rm e}i_{\rm e} = (-32)(150\times10^{-6}) = -4.8\ \text{mW}\\
p_{\rm f} &= -v_{\rm f}i_{\rm f} = -(60)(-350\times10^{-6}) = 21\ \text{mW}\\
p_{\rm g} &= v_{\rm g}i_{\rm g} = (-48)(-200\times10^{-6}) = 9.6\ \text{mW}\\
p_{\rm h} &= v_{\rm h}i_{\rm h} = (80)(-150\times10^{-6}) = -12\ \text{mW}\\
p_{\rm j} &= -v_{\rm j}i_{\rm j} = -(80)(-300\times10^{-6}) = 24\ \text{mW}
\end{aligned}
$$

Therefore,

$$\sum P_{\text{abs}} = 21 + 9.6 + 24 = 54.6\ \text{mW}$$

$$\sum P_{\text{del}} = 9 + 11 + 7 + 10.8 + 4.8 + 12 = 54.6\ \text{W}$$

$$\sum P_{\text{abs}} = \sum P_{\text{del}}$$

Thus, the interconnection satisfies the power check

P 1.30 [a] From an examination of reference polarities, elements a, b, e, and f absorb power, while elements c, d, g, and h supply power.

[b]

$$
\begin{aligned}
p_{\rm a} &= v_{\rm a}i_{\rm a} = (0.300)(25\times10^{-6}) = 7.5\,\mu\text{W}\\
p_{\rm b} &= -v_{\rm b}i_{\rm b} = -(-0.100)(10\times10^{-6}) = 1\,\mu\text{W}\\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (-0.200)(15\times10^{-6}) = -3\,\mu\text{W}\\
p_{\rm d} &= -v_{\rm d}i_{\rm d} = -(-0.200)(-35\times10^{-6}) = -7\,\mu\text{W}\\
p_{\rm e} &= -v_{\rm e}i_{\rm e} = -(0.350)(-25\times10^{-6}) = 8.75\,\mu\text{W}\\
p_{\rm f} &= v_{\rm f}i_{\rm f} = (0.200)(10\times10^{-6}) = 2\,\mu\text{W}\\
p_{\rm g} &= v_{\rm g}i_{\rm g} = (-0.250)(35\times10^{-6}) = -8.75\,\mu\text{W}\\
p_{\rm h} &= v_{\rm h}i_{\rm h} = (0.050)(-10\times10^{-6}) = -0.5\,\mu\text{W}
\end{aligned}
$$

$\sum P_{\text{abs}} = 7.5 + 1 + 8.75 + 2 = 19.25\,\mu\text{W}$

$\sum P_{\text{del}} = 3 + 7 + 8.75 + 0.5 = 19.25\,\mu\text{W}$

Thus, 19.25 μW of power is delivered and 19.25 μW of power is absorbed, and the power balances