EGR 260
Circuit Analysis
File: N260H1S

Solution to Homework Assignment #1

Problem Assignment:

1) Chapter 1 # 8, 9, 11, 12, 15, 16, 17, 18, 19, 22, 26, 29
2) Using the power rating values from Table 1 (or from the Lecture #2 notes), calculate the following if the cost of electricity is $0.08/kW·h:
   A) The cost to run an air conditioner 12 h/day for 30 days
   B) The cost to watch a television for 4 h/day for 1 year
   C) The cost to dry your hair with a blow dryer (10 minutes)

Solutions:

P 1.8 $$n = \frac{35 \times 10^{-6}\ \text{C/s}}{1.6022 \times 10^{-19}\ \text{C/elec}} = 2.18 \times 10^{14}\ \text{elec/s}$$

P 1.9 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 24\cos 4000t$$

Therefore, $dq = 24\cos 4000t\, dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 24\int_0^t \cos 4000y\, dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 24\frac{\sin 4000y}{4000}\Big|_0^t = \frac{24}{4000}\sin 4000t - \frac{24}{4000}\sin 4000(0) = \frac{24}{4000}\sin 4000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6 \times 10^{-3}\sin 4000t\ \text{C} = 6\sin 4000t\ \text{mC}$

P 1.11 $p = (9)(100 \times 10^{-3}) = 0.9\ \text{W}$; $\quad 5\ \text{hr} \cdot \frac{3600\ \text{s}}{1\ \text{hr}} = 18{,}000\ \text{s}$

$$w(t) = \int_0^t p\, dt \qquad w(18{,}000) = \int_0^{18{,}000} 0.9\, dt = 0.9(18{,}000) = 16.2\ \text{kJ}$$

P 1.12 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = vi$, since the current i is flowing into the + terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (120)(5) = 600$ W 600 W from A to B

[b] $p = (250)(-8) = -2000$ W 2000 W from B to A

[c] $p = (-150)(16) = -2400$ W 2400 W from B to A

[d] $p = (-480)(-10) = 4800$ W 4800 W from A to B

P 1.15 [a] In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,

$p = vi = (30)(12) = 360$ W.

Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

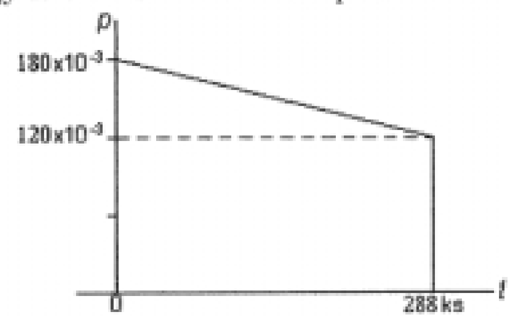
[b] $w(t) = \int_0^t p\,dx; \qquad 1\text{ min} = 60\text{ s}$

$$w(60) = \int_0^{60} 360\,dx$$

$$w = 360(60 - 0) = 360(60) = 21{,}600\text{ J} = 21.6\text{ kJ}$$

P 1.16 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 80 hr = 288,000 s = 288 ks

$$p(0) = (9)(20 \times 10^{-3}) = 180 \times 10^{-3}\text{ W}$$

$$p(288\text{ ks}) = (6)(20 \times 10^{-3}) = 120 \times 10^{-3}\text{ W}$$

$$w = (120 \times 10^{-3})(288 \times 10^3) + \frac{1}{2}(180 \times 10^{-3} - 120 \times 10^{-3})(288 \times 10^3) = 43.2\text{ kJ}$$

P 1.17 [a] $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$

$p(1\text{ ms}) = 3.1\text{ mW}$

[b]
$$\begin{aligned} w(t) &= \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + \\ &\quad 50e^{-2000x} - 10e^{-3000x})dx \\ &= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - \\ &\quad 25e^{-2000t} + 3.33e^{-3000t}\mu\text{J} \end{aligned}$$

$$w(1\text{ ms}) = 1.24\mu\text{J}$$

[c] $w_{\text{total}} = 21.67\mu\text{J}$

P 1.18 [a] $v(10\text{ ms}) = 400e^{-1}\sin 2 = 133.8\text{ V}$

$i(10\text{ ms}) = 5e^{-1}\sin 2 = 1.67\text{ A}$

$p(10\text{ ms}) = vi = 223.79\text{ W}$

[b]
$$\begin{aligned} p &= vi = 2000e^{-200t}\sin^2 200t \\ &= 2000e^{-200t}\left[\frac{1}{2} - \frac{1}{2}\cos 400t\right] \\ &= 1000e^{-200t} - 1000e^{-200t}\cos 400t \\ w &= \int_0^\infty 1000e^{-200t}\,dt - \int_0^\infty 1000e^{-200t}\cos 400t\,dt \\ &= 1000\left.\frac{e^{-200t}}{-200}\right|_0^\infty \\ &\quad -1000\left.\left\{\frac{e^{-200t}}{(200)^2 + (400)^2}[-200\cos 400t + 400\sin 400t]\right\}\right|_0^\infty \\ &= 5 - 1000\left[\frac{200}{4\times 10^4 + 16\times 10^4}\right] = 5 - 1 \\ \text{w} &= 4\text{ J} \end{aligned}$$

P 1.19 [a] $0\text{ s} \le t < 4\text{ s}$:

$v = 2.5t$ V; $i = 1\,\mu$A; $p = 2.5t\,\mu$W

$4\text{ s} < t \le 8\text{ s}$:

$v = 10$ V; $i = 0$ A; $p = 0$ W

$8\text{ s} \le t < 16\text{ s}$:

$v = -2.5t + 30$ V; $i = -1\,\mu$A; $p = 2.5t - 30\,\mu$W

$16\text{ s} < t \le 20\text{ s}$:

$v = -10$ V; $i = 0$ A; $p = 0$ W

$20\text{ s} \le t < 36\text{ s}$:

$v = t - 30$ V; $i = 0.4\,\mu$A; $p = 0.4t - 12\,\mu$W

$36\text{ s} < t \le 46\text{ s}$:

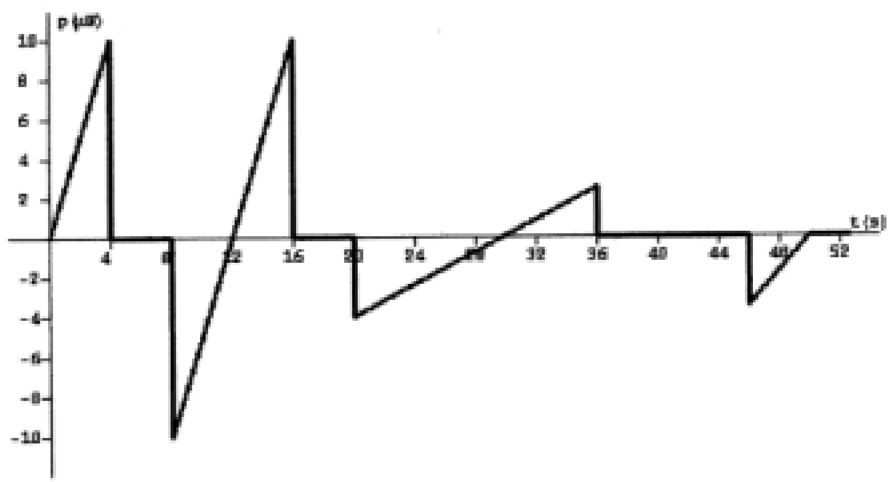
$v = 6$ V; $i = 0$ A; $p = 0$ W

$46\text{ s} \le t < 50\text{ s}$:

$v = -1.5t + 75$ V; $i = -0.6\,\mu$A; $p = 0.9t - 45\,\mu$W

$t > 50$ s:

$v = 0$ V; $i = 0$ A; $p = 0$ W



[b] Calculate the area under the curve from zero up to the desired time:

$$w(4) = \tfrac{1}{2}(4)(10) = 20\,\mu\text{J}$$

$$w(12) = w(4) - \tfrac{1}{2}(4)(10) = 0\text{ J}$$

$$w(36) = w(12) + \tfrac{1}{2}(4)(10) - \tfrac{1}{2}(10)(4) + \tfrac{1}{2}(6)(2.4) = 7.2\,\mu\text{J}$$

$$w(50) = w(36) - \tfrac{1}{2}(4)(3.6) = 0\text{ J}$$

P 1.22 [a]

$$q = \text{area under } i \text{ vs. } t \text{ plot}$$
$$= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right] \times 10^3$$
$$= [10 + 40 + 16 + 48 + 9]10^3 = 123{,}000 \text{ C}$$

[b]

$$w = \int p\,dt = \int vi\,dt$$
$$v = 0.2 \times 10^{-3}t + 9 \qquad 0 \le t \le 15 \text{ ks}$$

$0 \le t \le 4000s$

$$i = 15 - 1.25 \times 10^{-3}t$$
$$p = 135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2$$
$$w_1 = \int_0^{4000} (135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2)\,dt$$
$$= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ}$$

$4000 \le t \le 12{,}000$

$$i = 12 - 0.5 \times 10^{-3}t$$
$$p = 108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2$$
$$w_2 = \int_{4000}^{12{,}000} (108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2)\,dt$$
$$= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ}$$

$12{,}000 \le t \le 15{,}000$

$$i = 30 - 2 \times 10^{-3}t$$
$$p = 270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2$$
$$w_3 = \int_{12{,}000}^{15{,}000} (270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2)\,dt$$
$$= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ}$$
$$w_T = w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ}$$

2A) Since power is constant, W = P*t = (860W)(12 h/day)(30 days) = 309.6 kW·h
Cost = (309.6 kW·h)($0.08/kW·h) = $24.77

$2B) Since power is constant, W = P*t = (150W)(4 h/day)(365 days) = 219 kW·h
Cost = (219 kW·h)($0.08/kW·h) = $17.52

2C) Since power is constant, W = P*t = (1300W)(10 min)(1h/60 min) = 0.2667 kW·h
Cost = (0.26667 kW·h)($0.08/kW·h) = $0.017

P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$p_a = v_a i_a = (0.150)(0.6) = 90 \text{ mW}$$
$$p_b = v_b i_b = (0.150)(-1.4) = -210 \text{ mW}$$
$$p_c = -v_c i_c = -(0.100)(-0.8) = 80 \text{ mW}$$
$$p_d = v_d i_d = (0.250)(-0.8) = -200 \text{ mW}$$
$$p_e = -v_e i_e = -(0.300)(-2) = 600 \text{ mW}$$
$$p_f = v_f i_f = (-0.300)(1.2) = -360 \text{ mW}$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:

$\sum P_{dev} = 210 + 200 + 360 = 770$ mW;

$\sum P_{abs} = 90 + 80 + 600 = 770$ mW

Thus, the power balances and the total power developed in the circuit is 770 mW.

P 1.29

$$p_a = -v_a i_a = -(1.6)(0.080) = -128 \text{ mW}$$
$$p_b = -v_b i_b = -(2.6)(0.060) = -156 \text{ mW}$$
$$p_c = v_c i_c = (-4.2)(-0.050) = 210 \text{ mW}$$
$$p_d = -v_d i_d = -(1.2)(0.020) = -24 \text{ mW}$$
$$p_e = v_e i_e = (1.8)(0.030) = 54 \text{ mW}$$
$$p_f = -v_f i_f = -(-1.8)(-0.040) = -72 \text{ mW}$$
$$p_g = v_g i_g = (-3.6)(-0.030) = 108 \text{ mW}$$
$$p_h = v_h i_h = (3.2)(-0.020) = -64 \text{ mW}$$
$$p_j = -v_j i_j = -(-2.4)(0.030) = 72 \text{ mW}$$

$\sum P_{del} = 128 + 156 + 24 + 72 + 64 = 444$ mW

$\sum P_{abs} = 210 + 54 + 108 + 72 = 444$ mW

Therefore, $\sum P_{del} = \sum P_{abs} = 444$ mW

Thus, the interconnection satisfies the power check