EGR 260 - Circuit Analysis
File: N260H4BS

Solution to HW #4 – Nodal Analysis

Problem Assignment:
Nodal analysis problems: 9, 10a, 12, 14, 19, 21, 22, 4.31a (using node equations)

P 4.9 $$2.4 + \frac{v_1}{125} + \frac{v_1 - v_2}{25} = 0$$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{250} + \frac{v_2}{375} - 3.2 = 0$$

Solving, $v_1 = 25$ V; $\qquad v_2 = 90$ V

CHECK:

$$p_{125\Omega} = \frac{(25)^2}{125} = 5 \text{ W}$$

$$p_{25\Omega} = \frac{(90 - 25)^2}{25} = 169 \text{ W}$$

$$p_{250\Omega} = \frac{(90)^2}{250} = 32.4 \text{ W}$$

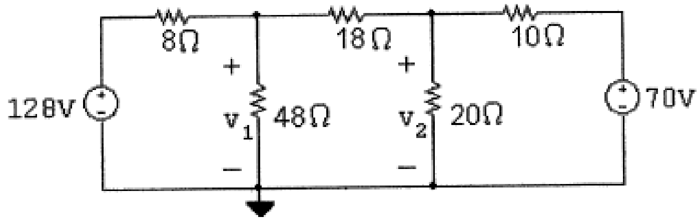
$$p_{375\Omega} = \frac{(90)^2}{375} = 21.6 \text{ W}$$

$$p_{2.4\text{A}} = (25)(2.4) = 60 \text{ W}$$

$$\sum p_{\text{abs}} = 5 + 169 + 32.4 + 21.6 + 60 = 288 \text{ W}$$

$$\sum p_{\text{dev}} = (90)(3.2) = 288 \text{ W} \quad \text{(CHECKS)}$$

P 4.10 [a]



$$\frac{v_1 - 128}{8} + \frac{v_1}{48} + \frac{v_1 - v_2}{18} = 0$$
$$\frac{v_2 - v_1}{18} + \frac{v_2}{20} + \frac{v_2 - 70}{10} = 0$$

In standard form,

$$v_1\left(\frac{1}{8} + \frac{1}{48} + \frac{1}{18}\right) + v_2\left(-\frac{1}{18}\right) = \frac{128}{8}$$
$$v_1\left(-\frac{1}{18}\right) + v_2\left(\frac{1}{18} + \frac{1}{20} + \frac{1}{10}\right) = \frac{70}{10}$$

Solving, $v_1 = 96$ V; $\quad v_2 = 60$ V

$$i_a = \frac{128 - 96}{8} = 4 \text{ A}$$

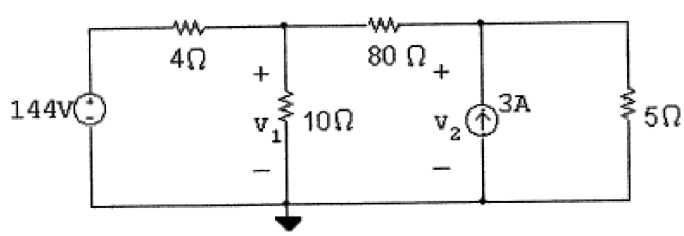
$$i_b = \frac{96}{48} = 2 \text{ A}$$

$$i_c = \frac{96 - 60}{18} = 2 \text{ A}$$

$$i_d = \frac{60}{20} = 3 \text{ A}$$

$$i_e = \frac{60 - 70}{10} = -1 \text{ A}$$

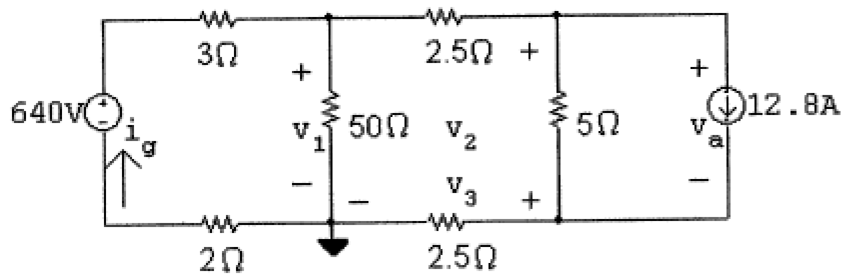
P 4.12



$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$
$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100$ V; $\qquad v_2 = 20$ V

P 4.14 [a]



$$\frac{v_1}{50} + \frac{v_1 - 640}{5} + \frac{v_1 - v_2}{2.5} = 0 \qquad \text{so} \qquad 31v_1 - 20v_2 + 0v_3 = 6400$$

$$\frac{v_2 - v_1}{2.5} + \frac{v_2 - v_3}{5} + 12.8 = 0 \qquad \text{so} \qquad -2v_1 + 3v_2 - v_3 = -64$$

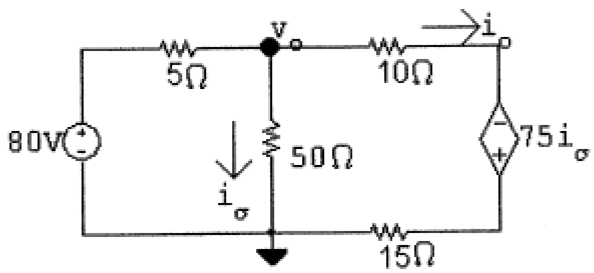
$$\frac{v_3}{2.5} + \frac{v_3 - v_2}{5} - 12.8 = 0 \qquad \text{so} \qquad 0v_1 - v_2 + 3v_3 = 64$$

Solving, $v_1 = 380\ V$; $v_2 = 269$ V; $v_3 = 111$ V,

[b] $i_g = \dfrac{640 - 380}{5} = 52$ A

$p_g(\text{del}) = (640)(52) = 33{,}280$ W

P 4.19



$$\frac{v_o - 80}{5} + \frac{v_o}{50} + \frac{v_o + 75i_\sigma}{25} = 0; \quad i_\sigma = \frac{v_o}{50}$$

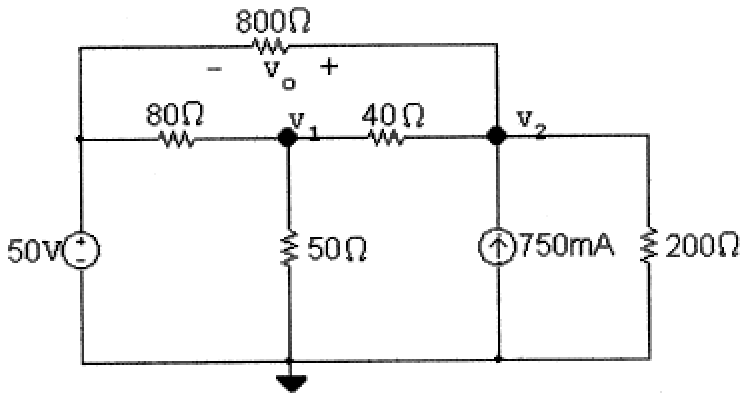
Solving, $v_o = 50$ V; $i_\sigma = 1$ A

$$i_o = \frac{50 - (-75)(1)}{25} = 5 \text{ A}$$

$p_{75i_\sigma} = 75 i_\sigma i_o = -375$ W

∴ The dependent voltage source delivers 375 W to the circuit.

P 4.21



The two node voltage equations are:

$$\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40}\right) + v_2\left(-\frac{1}{40}\right) = \frac{50}{80}$$

$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800}\right) = 0.75 + \frac{50}{800}$$

Solving, $v_1 = 34$ V; $v_2 = 53.2$ V.
Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2$ V.
POWER CHECK:

$$i_g = (50 - 34)/80 + (50 - 53.2)/800 = 196 \text{ m A}$$

$$p_{50\text{V}} = -(50)(0.196) = -9.8 \text{ W}$$

$$p_{80\Omega} = (50 - 34)^2/80 = 3.2 \text{ W}$$

$$p_{800\Omega} = (50 - 53.2)^2/800 = 12.8 \text{ m W}$$

$$p_{40\Omega} = (53.2 - 34)^2/40 = 9.216 \text{ W}$$

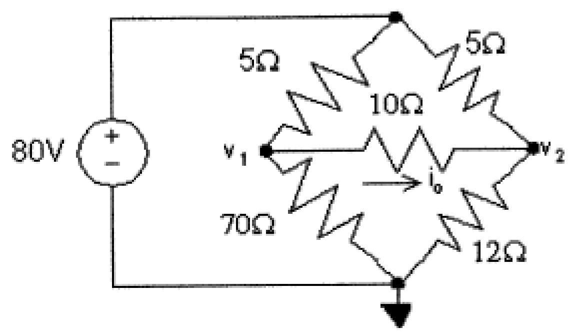
$$p_{50\Omega} = 34^2/50 = 23.12 \text{ W}$$

$$p_{200\Omega} = 53.2^2/200 = 14.1512 \text{ W}$$

$$p_{0.75\text{A}} = -(53.2)(0.75) = -39.9 \text{ W}$$

$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7 \text{ W} = \sum p_{\text{del}} =$
$9.8 + 39.9 = 49.7$

P 4.22



$$\frac{v_1}{70} + \frac{v_1 - v_2}{10} + \frac{v_1 - 80}{5} = 0 \qquad \text{so} \qquad 22v_1 - 7v_2 = 1120$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{10} + \frac{v_2 - 80}{5} = 0 \qquad \text{so} \qquad -6v_1 + 23v_2 = 960$$

Solving, $v_1 = 70$ V; $\qquad v_2 = 60$ V

Thus, $i_o = \dfrac{v_1 - v_2}{10} = 1$ A

4.31a) Solve using node equations (to be added later)