EGR 260
Circuit Analysis
File: N260H8BS

Solution to Homework Assignment #8

Chapter 6 in **Electric Circuits, 8th Edition** by Nilsson
Ch. 6 problems: 15, 17, 18, 19, 25, 26, 27

P 6.15 [a] $0 \le t \le 100\,\mu s$

$$C = 0.2\,\mu F \qquad \frac{1}{C} = 5 \times 10^6$$

$$v = 5 \times 10^6 \int_0^t -0.04\,dx + 40$$

$$v = -200 \times 10^3 t + 40\,V \qquad 0 \le t \le 100\,\mu s$$

$$v(100\,\mu s) = -20 + 40 = 20\,V$$

[b] $100\,\mu s \le t \le 300\,\mu s$

$$v = 5 \times 10^6 \int_{100\times10^{-6}}^t 0.08\,dx + 20 = 4 \times 10^5 t - 40 + 20$$

$$v = 4 \times 10^5 t - 20V \qquad 100 \le t \le 300\,\mu s$$

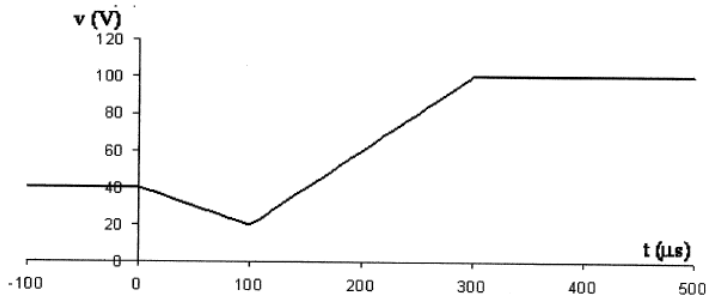
$$v(300\,\mu s) = 4 \times 10^5 (300 \times 10^{-6}) - 20 = 100\,V$$

[c] $300\,\mu s \le t < \infty$

$$v = 5 \times 10^6 \int_{300\times10^{-6}}^t 0\,dx + 100 = 100$$

$$v = 100\,V, \qquad 300\,\mu s \le t < \infty$$

[d]



P 6.17 [a] $i = \dfrac{50 \times 10^{-3}}{10 \times 10^{-6}} t = 5 \times 10^3 t \qquad 0 \le t \le 10\,\mu s$

$$i = 50 \times 10^{-3} \qquad 10 \le t \le 30\,\mu s$$

$$q = \int_0^{10\times10^{-6}} 5 \times 10^3 t\,dt + \int_{10\times10^{-6}}^{30\times10^{-6}} 50 \times 10^{-3}\,dt$$

$$= 5 \times 10^3 \frac{t^2}{2}\Big|_0^{10\times10^{-6}} + 50 \times 10^{-3}(20 \times 10^{-6})$$

$$= 5 \times 10^3 (\tfrac{1}{2})(100 \times 10^{-12}) + 1000 \times 10^{-3} \times 10^{-6}$$

$$= 1.25\,\mu C$$

[b] $i = 200 \times 10^{-3} - 5 \times 10^{-3} t \qquad 30\,\mu s \le t \le 50\,\mu s$

$$q = 1.25 \times 10^{-6} + \int_{30\times10^{-6}}^{50\times10^{-6}} [200 \times 10^{-3} - 5 \times 10^3 t]\,dt$$

$$= 1.25 \times 10^{-6} + 200 \times 10^{-3}(20 \times 10^{-6}) - 5 \times 10^3 \frac{t^2}{2}\Big|_{30\times10^{-6}}^{50\times10^{-6}}$$

$$= 1.25 \times 10^{-6} + 4000 \times 10^{-9} - 5 \times 10^3 \left[\frac{2500 - 900}{2}\right] 10^{-12}$$

$$= 1.25\,\mu C$$

Since $q = vC$, $\quad \therefore \quad v = 1.25/0.25 = 5\,V.$

[c] $i = -300 \times 10^{-3} + 5 \times 10^{-3} t \qquad 50\,\mu s \le t \le 60\,\mu s$

$$q = 1.25 \times 10^{-6} + \int_{50\times10^{-6}}^{60\times10^{-6}} [-300 \times 10^{-3} + 5 \times 10^3 t]\,dt$$

$$= 1.25 \times 10^{-6} - 300 \times 10^{-3}(10 \times 10^{-6})$$

$$+5 \times 10^3 \left[\frac{3600 - 2500}{2}\right] 10^{-12}$$

$$= 1\,\mu C$$

$$v = \frac{1 \times 10^{-6}}{0.25 \times 10^{-6}} = 4\,V$$

$$w = \frac{C}{2} v^2 = \frac{1}{2}(0.25) \times 10^{-6}(16) = 2\,\mu J$$

P 6.18 **[a]** $$v = 5\times10^{6}\int_{0}^{250\times10^{-6}} 100\times10^{-3}e^{-1000t}\,dt - 60.6$$

$$= 500\times10^{3}\frac{e^{-1000t}}{-1000}\Big|_{0}^{250\times10^{-6}} - 60.6$$

$$= 500(1-e^{-0.25}) - 60.6 = 50\,\text{V}$$

$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.2)(10^{-6})(50)^2 = 250\,\mu\text{J}$$

[b] $v = 500 - 60.6 = 439.40\,\text{V}$

$$w = \frac{1}{2}(0.2)\times10^{-6}(439.40)^2 = 19.31\,\text{mJ} = 19{,}307.24\,\mu\text{J}$$

P 6.19 **[a]** $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.40)\times10^{-6}(25)^2 = 125\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-1500t}$

$v(0) = A_2 = 25\,\text{V}$

$$\frac{dv}{dt} = -1500e^{-1500t}(A_1t + A_2) + e^{-1500t}(A_1)$$

$$= (-1500A_1t - 1500A_2 + A_1)e^{-1500t}$$

$$\frac{dv}{dt}(0) = A_1 - 1500A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore \quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{90\times10^{-3}}{0.40\times10^{-6}} = 225\times10^{3}$$

$$\therefore \quad 225\times10^{3} = A_1 - 1500(25)$$

Thus, $A_1 = 2.25\times10^{5} + 3.75\times10^{4} = 262{,}500\,\frac{\text{V}}{\text{s}}$

[c] $v = (262{,}500t + 25)e^{-1500t}$

$$i = C\frac{dv}{dt} = 0.40\times10^{-6}\frac{d}{dt}(262{,}500t + 25)e^{-1500t}$$

$$i = \frac{d}{dt}[(0.105t + 10\times10^{-6})e^{-1500t}]$$

$$= (0.105t + 10\times10^{-6})(-1500)e^{-1500t} + e^{-1500t}(0.105)$$

$$= (-157.5t - 15\times10^{-3} + 0.105)e^{-1500t}$$

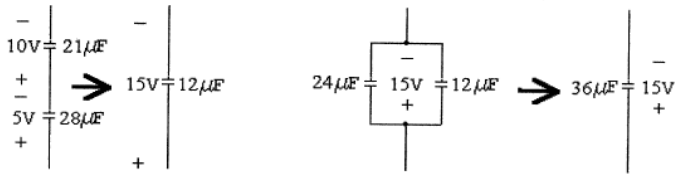
$$= (0.09 - 157.5t)e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$= (90 - 157{,}500t)e^{-1500t}\,\text{mA}, \qquad t \geq 0$$

P 6.25 $\frac{1}{21}+\frac{1}{28}=\frac{7}{84} \quad \therefore \quad C_{eq}=12\,\mu\text{F}$

$-10\,\text{V}-5\,\text{V}=-15\,\text{V}$

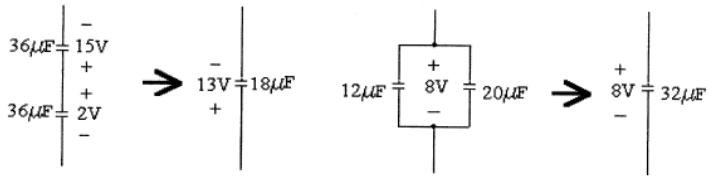
$24+12=36\,\mu\text{F}$



$\frac{1}{36}+\frac{1}{36}=\frac{2}{36} \quad \therefore \quad C_{eq}=18\,\mu\text{F}$

$-15\,\text{V}+2\,\text{V}=-13\,\text{V}$

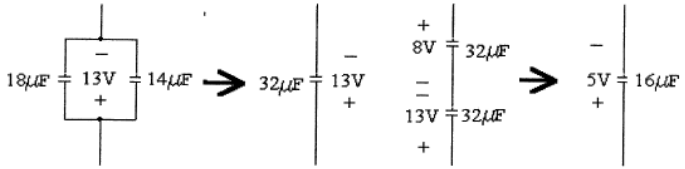
$12+20=32\,\mu\text{F}$



$18+14=32\,\mu\text{F}$

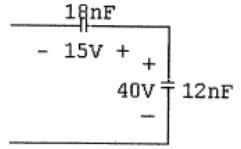
$\frac{1}{32}+\frac{1}{32}=\frac{2}{32} \quad \therefore \quad C_{eq}=16\,\mu\text{F}$

$8\,\text{V}-13\,\text{V}=-5\,\text{V}$



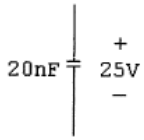
P 6.26 $\frac{1}{C_1}=\frac{1}{8}+\frac{1}{32}=\frac{5}{32}; \qquad C_1=6.4\,\text{nF}$

$C_2=5.6+6.4=12\,\text{nF}$

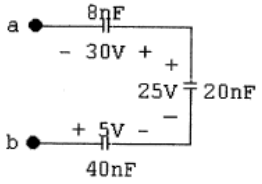


$\frac{1}{C_3}=\frac{1}{18}+\frac{1}{12}=\frac{10}{72}; \qquad C_3=7.2\,\text{nF}$

$C_4=12.8+7.2=20\,\text{nF}$

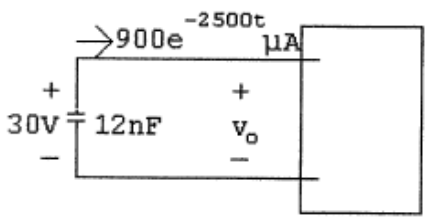


$\frac{1}{C_5}=\frac{1}{8}+\frac{1}{20}+\frac{1}{40}=\frac{1}{5}; \qquad C_5=5\,\text{nF}$



Equivalent capacitance is 5 nF with an initial voltage drop of −10 V.

P 6.27 [a]



$$v_o = -\frac{10^9}{12}\int_0^t 900\times10^{-6}e^{-2500x}\,dx + 30$$

$$= -75{,}000\frac{e^{-2500x}}{-2500}\Big|_0^t + 30$$

$$= 30e^{-2500t}\,\text{V}, \qquad t \geq 0$$

[b] $$v_1 = -\frac{10^9}{20}(900\times10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t + 45$$

$$= 18e^{-2500t} + 27\,\text{V}, \qquad t \geq 0$$

[c] $$v_2 = -\frac{10^9}{30}(900\times10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t - 15$$

$$= 12e^{-2500t} - 27\,\text{V}, \qquad t \geq 0$$

[d] $$p = vi = (30e^{-2500t})(900\times10^{-6})e^{-2500t}$$

$$= 27\times10^{-3}e^{-5000t}$$

$$w = \int_0^\infty 27\times10^{-3}e^{-5000t}\,dt$$

$$= 27\times10^{-3}\frac{e^{-5000t}}{-5000}\Big|_0^\infty$$

$$= -5.4\times10^{-6}(0-1) = 5.4\,\mu\text{J}$$

[e] $$w = \tfrac{1}{2}(20\times10^{-9})(45)^2 + \tfrac{1}{2}(30\times10^{-9})(15)^2$$

$$= 20.25\times10^{-6} + 3.375\times10^{-6}$$

$$= 23.625\,\mu\text{J}$$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 23.625 - 5.4 = 18.225\,\mu\text{J}$

[g] $$w_{\text{trapped}} = \tfrac{1}{2}(20\times10^{-9})(27)^2 + \tfrac{1}{2}(30\times10^{-9})(27)^2$$

$$= (10+15)(27)^2\times10^{-9}$$

$$= 18.225\,\mu\text{J}$$

CHECK: $18.225 + 5.4 = 23.625\,\mu\text{J}$