EGR 260
Circuit Analysis
File: N260H9BS

# Solution to Homework Assignment #9

Chapter 6 in **Electric Circuits, 8th Edition** by Nilsson
Ch. 6 problems: 1, 2, 5, 8, 21, 22, 33

P 6.1 [a]

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 16t\,\text{A} && 0 \le t \le 25\,\text{ms} \\ i &= 0.8 - 16t\,\text{A} && 25 \le t \le 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 375 \times 10^{-3}(16) = 6\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$v = 375 \times 10^{-3}(-16) = -6\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 6\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -6\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 && t < 0 \\ p &= 96t\,\text{W} && 0 < t < 25\,\text{ms} \\ p &= 96t - 4.8\,\text{W} && 25 < t < 50\,\text{ms} \\ p &= 0 && 50\,\text{ms} < t \end{aligned}$$

$w = \dfrac{1}{2}Li^2$

$$\begin{aligned} w &= 0 && t < 0 \\ w &= 48t^2\,\text{J} && 0 < t < 25\,\text{ms} \\ w &= 48t^2 - 4.8t + 0.12\,\text{J} && 25 < t < 50\,\text{ms} \\ w &= 0 && 50\,\text{ms} < t \end{aligned}$$

P 6.2 [a] $0 \le t \le 1\,\text{ms}$ :

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{300}\int_0^t 6 \times 10^{-3}\,dx + 0$$

$$= 20x\Big|_0^t = 20t\,\text{A}$$

$1\,\text{ms} \le t \le 2\,\text{ms}$ :

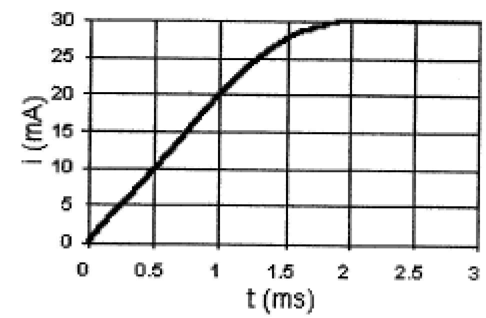
$$i = \frac{10^6}{300}\int_{10^{-3}}^t (12 \times 10^{-3} - 6x)\,dx + 20 \times 10^{-3}$$

$$\therefore \quad i = 40t - 10{,}000t^2 - 10 \times 10^{-3}\,\text{A}$$

$2\,\text{ms} \le t \le \infty$ :

$$i = \frac{10^6}{300}\int_{2\times 10^{-3}}^t (0)\,dx + 30 \times 10^{-3} = 30\,\text{mA}$$

[b]



P 6.5 [a] $0 \le t \le 2\,\text{s}$:

$$v = -25t$$

$$i = \frac{1}{2.5}\int_0^t -25x\,dx + 0 = -10\frac{x^2}{2}\Big|_0^t$$

$$i = -5t^2\ \text{A}$$

$2\,\text{s} \le t \le 6\,\text{s}$:

$$v = -100 + 25t$$

$$i(2) = -20\ \text{A}$$

$$\therefore\ i = \frac{1}{2.5}\int_2^t (25x - 100)\,dx - 20$$

$$= 10\int_2^t x\,dx - 40\int_2^t dx - 20$$

$$= 5(t^2 - 4) - 40(t - 2) - 20$$

$$= 5t^2 - 40t + 40\ \text{A}$$

$6\,\text{s} \le t \le 10\,\text{s}$:

$$v = 200 - 25t$$

$$i(6) = 5(36) - 240 + 40 = -20\ \text{A}$$

$$i = \frac{1}{2.5}\int_6^t (200 - 25x)\,dx - 20$$

$$= 80\int_6^t dx - 10\int_6^t x\,dx - 20$$

$$= 80(t - 6) - 10(t^2 - 36)/2 - 20 = 80t - 5t^2 - 320\ \text{A}$$

$10\,\text{s} \le t \le 12\,\text{s}$:

$$v = 25t - 300$$

$$i(10) = 800 - 500 - 320 = -20\ \text{A}$$

$$i = \frac{1}{2.5}\int_{10}^t (25x - 300)\,dx - 20$$

$$= 10\int_{10}^t x\,dx - 120\int_{10}^t dx - 20$$

$$= 5(t^2 - 100) - 120(t - 10) - 20$$

$$= 5t^2 - 120t + 680\ \text{A}$$

$t \ge 12\,\text{s}$:

$$v = 0$$

$$i(12) = 5(12)^2 - 120(12) + 680 = -40\ \text{A}$$

$$i = \frac{1}{2.5}\int_{12}^t 0\,dx - 40$$

$$= -40\ \text{A}$$

[b] For $0 \le t \le 2\,\text{s}$, $v = -25t$ V; $i = -5t^2$ A

$v = 0$ when $t = 0$ so $i = 0$ A

For $2 \le t \le 6\,\text{s}$, $v = -100 + 25t$ V; $i = 5t^2 - 40t + 40$ A

$v = 0$ when $t = 4\,\text{s}$ so $i = 5(4)^2 - 40(4) + 40 = -40$ A

For $6 \le t \le 10\,\text{s}$, $v = 200 - 25t$ V; $i = -5t^2 + 80t - 320$ A

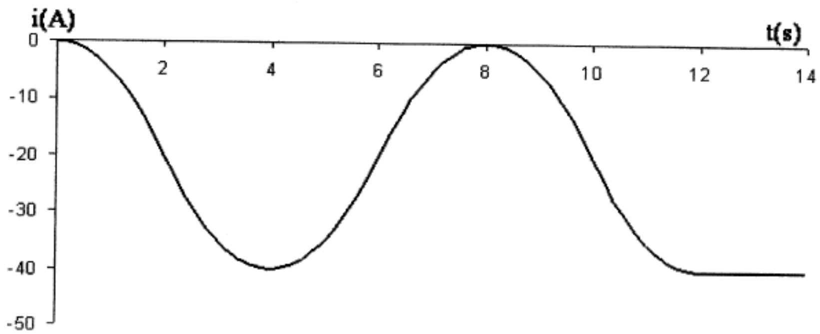
$v = 0$ when $t = 8\,\text{s}$ so $i = -5(8)^2 + 80(8) - 320 = 0$ A

For $10 \le t \le 12\,\text{s}$, $v = 25t - 300$ V; $i = 5t^2 - 120t + 680$ A

$v = 0$ when $t = 12\,\text{s}$ so $i = 5(12)^2 - 120(12) + 680 = -40$ A

For $t \ge 12\,\text{s}$, $v = 0$; $i = -40$ A

[c]




P 6.8 [a] $i(0) = A_1 + A_2 = 1$

$$\frac{di}{dt} = -2000A_1e^{-2000t} - 8000A_2e^{-8000t}$$

$$v = -30A_1e^{-2000t} - 120A_2e^{-8000t}\,\text{V}$$

$$v(0) = -30A_1 - 120A_2 = 60$$

Solving, $A_1 = 2$ and $A_2 = -1$

Thus,

$$i_1 = (2e^{-2000t} - e^{-8000t})\,\text{A} \qquad t \geq 0$$

$$v = -60e^{-2000t} + 120e^{-8000t}\,\text{V}, \qquad t \geq 0$$

[b] $p = vi = 300e^{-10{,}000t} - 120e^{-4000t} - 120e^{-16{,}000t}$

$p = 0$ when $300e^{6000t} - 120e^{12{,}000t} - 120 = 0$

Let $x = e^{6000t}$; then $300x - 120x^2 - 120 = 0$

Thus $x^2 - 2.5x + 1 = 0$ so $x = 0.5$ and $x = 2$

If $x = e^{6000t} = 0.5$, $t$ will be negative. Hence, the solution for $t > 0$ must be $x = 2$:

$e^{6000t} = 2$ so $6000t = \ln 2$

Thus, $t = \dfrac{\ln 2}{6000} = 115.52\,\mu\text{s}$

P 6.21 $6\|14 = 4.2\,\text{H}$

$$15.8 + 4.2 = 20\,\text{H}$$

$$20\|60 = 15\,\text{H}$$

$$15 + 5 = 20\,\text{H}$$

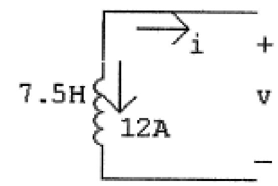
$$20\|80 = 16\,\text{H}$$

$$16 + 24 = 40\,\text{H}$$

$$40\|10 = 8\,\text{H}$$

$$L_{\text{ab}} = 12 + 8 = 20\,\text{H}$$

P 6.22 [a]




$$i(t) = -\frac{1}{7.5}\int_0^t -1800e^{-20x}\,dx - 12$$

$$= 240\frac{e^{-20x}}{-20}\Big|_0^t - 12$$

$$= -12(e^{-20t} - 1) - 12$$

$$i(t) = -12e^{-20t}\,\text{A}$$

[b] $$i_1(t) = -\frac{1}{10}\int_0^t -1800e^{-20x}\,dx + 4$$

$$= 180\frac{e^{-20x}}{-20}\Big|_0^t +4$$

$$= -9(e^{-20t} - 1) + 4$$

$$i_1(t) = -9e^{-20t} + 13\,\text{A}$$

[c] $$i_2(t) = -\frac{1}{30}\int_0^t -1800e^{-20x}\,dx - 16$$

$$= 60\frac{e^{-20x}}{-20}\Big|_0^t -16$$

$$= -3(e^{-20t} - 1) - 16$$

$$i_2(t) = -3e^{-20t} - 13\,\text{A}$$

[d] $p = vi = (-1800e^{-20t})(-12e^{-20t}) = 21{,}600e^{-40t}$ W

$$w = \int_0^\infty p\,dt = \int_0^\infty 21{,}600e^{-40t}\,dt$$

$$= 21{,}600\frac{e^{-40t}}{-40}\Big|_0^\infty$$

$$= 540\,\text{J}$$

[e] $w = \frac{1}{2}(10)(16) + \frac{1}{2}(30)(256) = 3920\,\text{J}$

[f] $w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 3920 - 540 = 3380\,\text{J}$

[g] $w_{\text{trapped}} = \frac{1}{2}(10)(13)^2 + \frac{1}{2}(30)(13)^2 = 3380\,\text{J}$ checks

P 6.33 $$v_c = \frac{-10^6}{20}\int_0^t e^{-80x}\sin 60x\,dx - 300$$

$$= 5e^{-80t}[80\sin 60t + 60\cos 60t] + 300 - 300$$

$$= 400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t \text{ V}$$

$$v_L = 5\frac{di_o}{dt}$$

$$= 5[-80e^{-80t}\sin 60t + 60e^{-80t}\cos 60t]$$

$$= -400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t \text{ V}$$

$$v_o = v_c - v_L$$

$$= (300e^{-80t}\cos 60t - 300e^{-80t}\cos 60t + 400e^{-80t}\sin 60t + 400e^{-80t}\sin 60t)$$

$$= 800e^{-80t}\sin 60t \text{ V}$$