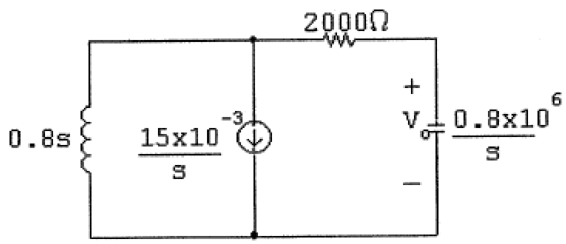
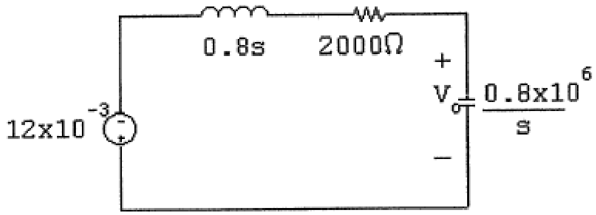
EGR 261
Signals and Systems
File: N261H2S

Solution to Homework Assignment #2

1) Work the following problems from Ch. 13 of *Electric Circuits, 8th Edition* by Nilsson: 9, 10, 13, 17, 27(a & c only), 44, 49, 51, 69a

P 13.9 [a] For $t > 0$:





[b] $$V_o = \frac{-12\times10^{-3}(0.8/s)\times10^6}{0.8s+2000+(0.8\times10^6)/s}$$

$$= \frac{-9600}{0.8s^2+2000s+0.8\times10^6}$$

$$= \frac{-12{,}000}{s^2+2500s+10^6}$$

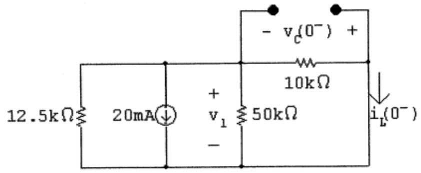
[c] $$V_o = \frac{-12{,}000}{(s+500)(s+2000)} = \frac{K_1}{s+500}+\frac{K_2}{s+2000}$$

$$K_1 = -8; \qquad K_2 = 8$$

$$V_o = \frac{-8}{s+500}+\frac{8}{s+2000}$$

$$v_o(t) = (-8e^{-500t}+8e^{-2000t})u(t)\,\text{V}$$

P 13.10 [a] For $t < 0$:



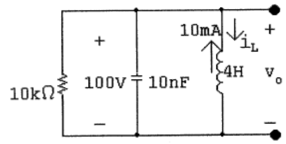
$$\frac{1}{R_e} = \frac{1}{12.5}+\frac{1}{50}+\frac{1}{10} = \frac{1}{5}; \qquad R_e = 5\,\text{k}\Omega$$

$v_1 = -20(5) = -100\,\text{V}$

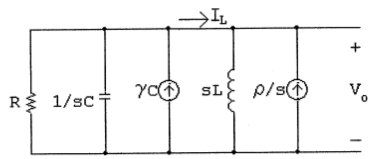
$$i_L(0^-) = \frac{-100}{10}\times10^{-3} = -10\,\text{mA}$$

$v_C(0^-) = -v_1 = 100\,\text{V}$

For $t = 0^+$:



s-domain circuit:



where

$R = 10\,\text{k}\Omega; \qquad C = 10\,\text{nF}; \qquad \gamma = 100\,\text{V};$

$L = 4\,\text{H}; \qquad \text{and} \qquad \rho = 10\,\text{mA}$

[b] $$\frac{V_o}{R}+V_o sC-\gamma C+\frac{V_o}{sL}-\frac{\rho}{s} = 0$$

$$\therefore\ V_o = \frac{\gamma[s+(\rho/\gamma C)]}{s^2+(1/RC)s+(1/LC)}$$

$$\frac{\rho}{\gamma C} = \frac{10\times10^{-3}}{(100)(10)10^{-9}} = 10^4$$

$$\frac{1}{RC} = \frac{10^9}{10^5} = 10^4$$

$$\frac{1}{LC} = \frac{10^9}{40} = 25 \times 10^6$$

$$V_o = \frac{100(s + 10^4)}{s^2 + 10^4 s + 25 \times 10^6}$$

[c] $I_L = \frac{V_o}{sL} - \frac{\rho}{s} = \frac{V_o}{4s} - \frac{10 \times 10^{-3}}{s}$

$$I_L = \frac{25(s + 10^4)}{s(s^2 + 10^4 s + 25 \times 10^6)} - \frac{10^{-2}}{s} = \frac{-0.01(s + 7500)}{(s + 5000)^2}$$

[d] $V_o = \frac{100(s + 10^4)}{s^2 + 10^4 s + 25 \times 10^6}$

$$= \frac{100(s + 10^4)}{(s + 5000)^2} = \frac{K_1}{(s + 5000)^2} + \frac{K_2}{s + 5000}$$

$$K_1 = 100(5000) = 5 \times 10^5$$

$$K_2 = \frac{d}{ds}\left[100(s + 10{,}000)\right]_{s=-5000} = 100$$

$$V_o = \frac{5 \times 10^5}{(s + 5000)^2} + \frac{100}{s + 5000}$$

$$v_o = [5 \times 10^5 t e^{-5000t} + 100 e^{-5000t}]u(t)\ \mathrm{V}$$

[e] $I_L = \frac{-0.01(s + 7500)}{(s + 5000)^2}$

$$= \frac{K_1}{(s + 5000)^2} + \frac{K_2}{(s + 5000)}$$

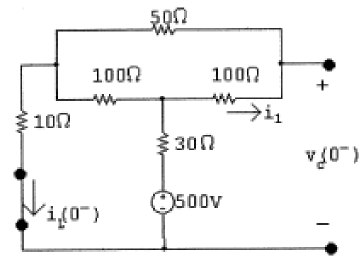
$$K_1 = -0.01(2500) = -25$$

$$K_2 = \frac{d}{ds}\left[-0.01(s + 7500)\right]_{s=-5000} = -0.01$$

$$I_L = \left[\frac{-25{,}000}{(s + 5000)^2} - \frac{10}{s + 5000}\right] \times 10^{-3}$$

$$i_L = -[25{,}000t + 10]e^{-5000t}u(t)\ \mathrm{mA}$$

P 13.13 [a] For $t < 0$:

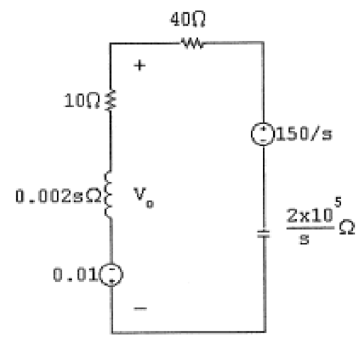


$$i_L(0^-) = \frac{500}{30 + 60 + 10} = 5\ \mathrm{A}$$

$$i_1 = \frac{5(100)}{250} = 2\ \mathrm{A}$$

$$v_C(0^-) = 500 - 5(30) - 2(100) = 500 - 350 = 150\ \mathrm{V}$$

For $t > 0$:



[b] $\frac{V_o + 0.01}{10 + 0.002s} + \frac{V_o - 150/s}{40 + 2 \times 10^5/s} = 0$

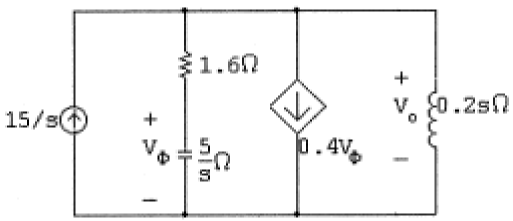
$$V_o\left[\frac{1}{10 + 0.002s} + \frac{s}{40s + 2 \times 10^5}\right] = \frac{150}{40s + 2 \times 10^5} - \frac{0.01}{10 + 0.002s}$$

$$V_o = \frac{-50(s + 5000)}{s^2 + 25{,}000s + 10^8} = \frac{-50(s + 5000)}{(s + 5000)(s + 20{,}000)}$$

$$V_o = \frac{-50}{s + 20{,}000}$$

[c] $v_o(t) = -50e^{-20{,}000t}u(t)\ \mathrm{V}$

P 13.17



$$\frac{15}{s} = \frac{V_o}{1.6+5/s} + 0.4V_\phi + \frac{V_o}{0.2s}$$

$$V_\phi = \frac{5/s}{1.6+5/s}V_o = \frac{5V_o}{1.6s+5}$$

$$\therefore \quad \frac{15}{s} = \frac{V_o s}{1.6s+5} + \frac{2V_o}{1.6s+5} + \frac{5V_o}{s}$$

$$= V_o\left[\frac{s(s+2)+5(1.6s+5)}{s(1.6s+5)}\right]$$

$$15(1.6s+5) = V_o(s^2+10s+25)$$

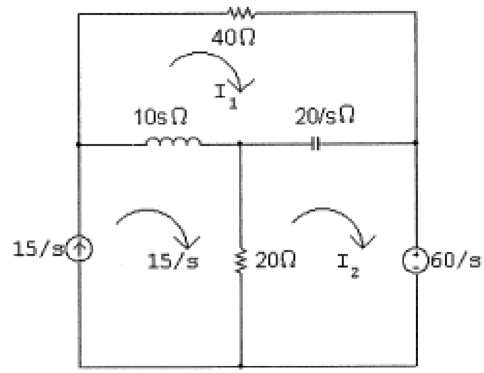
$$\therefore \quad V_o = \frac{15(1.6s+5)}{(s+5)^2} = \frac{K_1}{(s+5)^2} + \frac{K_2}{s+5}$$

$$K_1 = 15(-8+5) = -45; \qquad K_2 = 24$$

$$V_o = \frac{-45}{(s+5)^2} + \frac{24}{s+5}$$

$$v_o(t) = [-45te^{-5t} + 24e^{-5t}]u(t)\,\mathrm{V}$$

P 13.27 **[a]**



$$40I_1 + \frac{20}{s}(I_1 - I_2) + 10s(I_1 - 15/s) = 0$$

$$20(I_2 - 15/s) + \frac{20}{s}(I_2 - I_1) + \frac{60}{s} = 0$$

or

$$(s^2+4s+2)I_1 - 2I_2 = 15s$$

$$-I_1 + (s+1)I_2 = 12$$

$$\Delta = \begin{vmatrix} (s^2+4s+2) & -2 \\ -1 & (s+1) \end{vmatrix} = s(s+2)(s+3)$$

$$N_1 = \begin{vmatrix} 15s & -2 \\ 12 & (s+1) \end{vmatrix} = 15s^2 + 15s + 24$$

$$I_1 = \frac{N_1}{\Delta} = \frac{15s^2+15s+24}{s(s+2)(s+3)}$$

$$N_2 = \begin{vmatrix} (s^2+4s+2) & 15s \\ -1 & 12 \end{vmatrix} = 12s^2 + 63s + 24$$

$$I_2 = \frac{N_2}{\Delta} = \frac{12s^2+63s+24}{s(s+2)(s+3)}$$

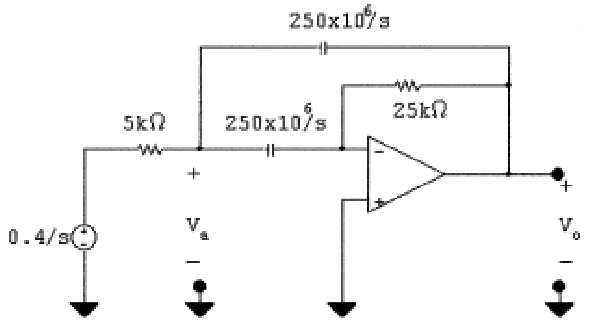
[c] $I_1 = \frac{4}{s} - \frac{27}{s+2} + \frac{38}{s+3}$

$$i_1(t) = (4 - 27e^{-2t} + 38e^{-3t})u(t)\text{ A}$$

$$I_2 = \frac{4}{s} + \frac{27}{s+2} - \frac{19}{s+3}$$

$$i_2(t) = (4 + 27e^{-2t} - 19e^{-3t})u(t)\text{ A}$$

P 13.44



$$\frac{V_a - 0.4/s}{5000} + \frac{V_a s}{250 \times 10^6} + \frac{(V_a - V_o)s}{250 \times 10^6} = 0$$

$$\frac{(0 - V_a)s}{250 \times 10^6} + \frac{(0 - V_o)}{25{,}000} = 0$$

$$V_a = \frac{-10^4 V_o}{s}$$

$$\therefore \quad V_o(s^2 + 20{,}000s + 500 \times 10^6) = -20{,}000$$

$$V_o = \frac{-20{,}000}{(s + 10{,}000 - j20{,}000)(s + 10{,}000 + j20{,}000)}$$

$$K_1 = \frac{-20{,}000}{j40{,}000} = j0.5 = 0.5\underline{/90^\circ}$$

$$v_o(t) = e^{-10{,}000t}\cos(20{,}000t + 90^\circ) = -e^{-10{,}000t}\sin(20{,}000t)u(t)\text{ V}$$

P 13.49 [a] $\frac{V_o}{V_i} = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs + 1}$

$$H(s) = \frac{(1/RC)}{s + (1/RC)} = \frac{50}{s + 50}; \qquad -p_1 = -50\text{ rad/s}$$

[b] $\frac{V_o}{V_i} = \frac{R}{R + 1/sC} = \frac{RCs}{RCs + 1} = \frac{s}{s + (1/RC)}$

$$= \frac{s}{s + 50}; \qquad z_1 = 0, \quad -p_1 = -50\text{ rad/s}$$

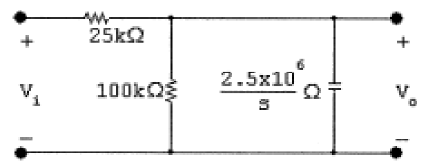
[c] $\frac{V_o}{V_i} = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 3 \times 10^6}$

$$z_1 = 0; \qquad -p_1 = -3 \times 10^6\text{ rad/s}$$

[d] $\frac{V_o}{V_i} = \frac{R}{R + sL} = \frac{R/L}{s + (R/L)} = \frac{3 \times 10^6}{s + 3 \times 10^6}$

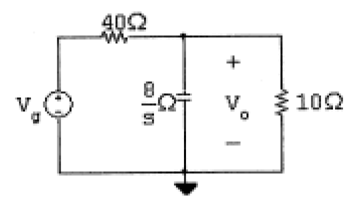
$$-p_1 = -3 \times 10^6\text{ rad/s}$$

[e]



$$\frac{V_o s}{2.5 \times 10^6} + \frac{V_o}{10^5} + \frac{V_o - V_i}{25 \times 10^3} = 0$$

P 13.69 [a]

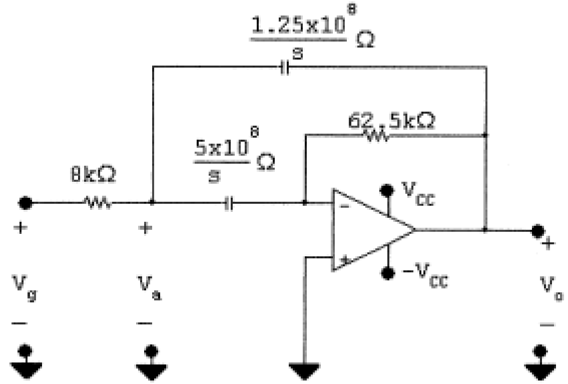


$$\frac{V_o - V_g}{40} + \frac{V_o s}{8} + \frac{V_o}{10} = 0$$

$$(5s + 5)V_o = V_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{0.2}{s + 1}; \qquad h(\lambda) = 0.2e^{-\lambda}u(\lambda)$$

P 13.51 [a]



$$\frac{V_a - V_g}{8000} + \frac{V_a s}{5\times10^8} + \frac{(V_a - V_o)s}{1.25\times10^8} = 0$$

$$\frac{-V_a s}{5\times10^8} - \frac{V_o}{62{,}500} = 0; \qquad V_a = \frac{-8000V_o}{s}$$

$$\therefore \quad \frac{-8000V_o}{s}(5s + 62{,}500) - 4sV_o = 62{,}500V_g$$

$$\therefore \quad H(s) = \frac{V_o}{V_g} = \frac{-15{,}625s}{s^2 + 10{,}000s + 125\times10^6}$$

$$s_{1,2} = -5000 \pm \sqrt{25\times10^6 - 125\times10^6} = -5000 \pm j10{,}000$$

$$H(s) = \frac{-15{,}625s}{(s+5000-j10{,}000)(s+5000+j10{,}000)}$$

[b] $-p_1 = -5000 + j10{,}000$ rad/s

$-p_2 = -5000 - j10{,}000$ rad/s

$z = 0$

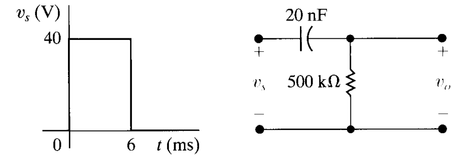
2) Use Laplace transforms to solve problem 7.81 of *Electric Circuits,8 E* by Nilsson.

7.81 PSPICE The voltage waveform shown in Fig. P7.81(a) is applied to the circuit of Fig. P7.81(b). The initial voltage on the capacitor is zero.

a) Calculate $v_o(t)$.

b) Make a sketch of $v_o(t)$ versus t.

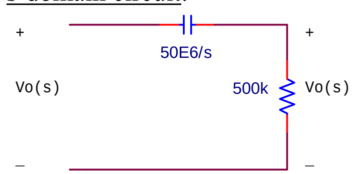
Figure P7.81



(a) (b)

Solution: $v_s(t) = 40[u(t) - u(t-0.006)]$ so $V_s(s) = \frac{40}{s} - \frac{40e^{-0.006s}}{s}$

s-domain circuit:



$$V_o(s) = V_s(s)\left[\frac{500k}{500k + \frac{50\times10^6}{s}}\right] = V_s(s)\left[\frac{s}{s+100}\right]$$

$$V_o(s) = \left[\frac{40}{s} - \frac{40e^{-0.006s}}{s}\right]\left[\frac{s}{s+100}\right] = \frac{40}{s+100} - \frac{40e^{-0.006s}}{s+100}$$

$$v_o(t) = 40e^{-100t}u(t) - 40e^{-100(t-0.006)}u(t-0.006)\ \mathrm{V}$$

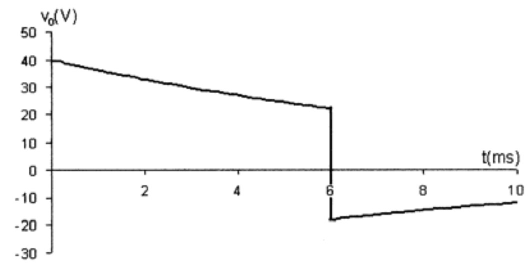
Sketch:

$v(0^+) = 40e^0 = 40$

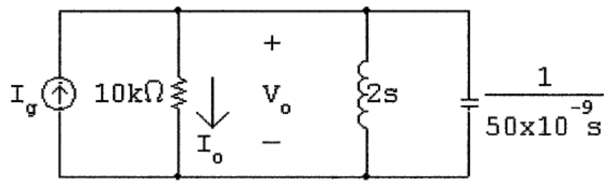
$v(6ms^-) = 40e^{-0.6} - 0 = 21.95$

$v(6ms^+) = 40e^{-0.6} - 40e^0 = -18.05$

$v(\infty) = 40e^{-\infty} - 40e^{-\infty} = 0$



P 13.55 [**a**]



$$\frac{V_o}{10{,}000} + \frac{V_o}{2s} + V_o(50 \times 10^{-9})s = I_g$$

$$\therefore \quad V_o = \frac{20 \times 10^6 s}{s^2 + 2000s + 10 \times 10^6} \cdot I_g$$

$$I_g = \frac{60 \times 10^{-3} s}{s^2 + 16 \times 10^6}; \qquad I_o = \frac{V_o}{10^4}$$

$$\therefore \quad H(s) = \frac{2000s}{s^2 + 2000s + 10^7}$$

3) Work problem 4.4-7 in *Linear Signals & Systems, 2nd Ed.* by Lathi:

4.4-7 Find the output voltage $y(t)$ for the network in Fig. P4.4-7 for the initial conditions $i_L(0) = 1$ A and $v_C(0) = 3$ V.

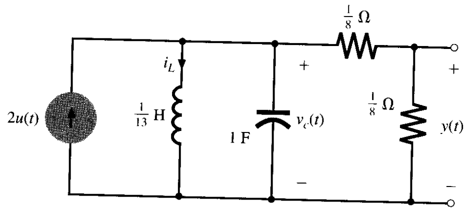
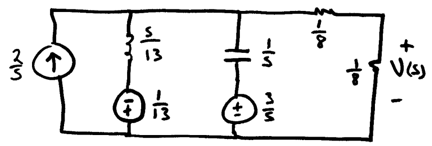
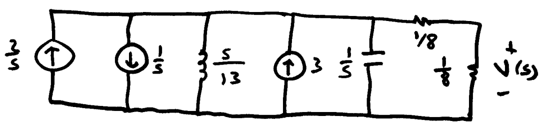


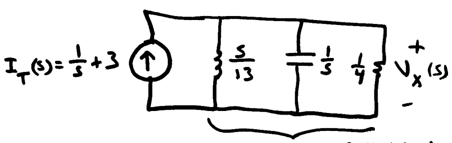
Figure P4.4-7

S-Domain CKT



After transforming the voltage sources:





Note: $\frac{V_x(s)}{2} = V(s)$

$$Z = \frac{s}{13} \| \frac{1}{s} \| \frac{1}{4} = \frac{s}{s^2+4s+13}$$

$$V_x(s) = I_T(s) \cdot Z = \left(\frac{1}{s}+3\right)\left(\frac{s}{s^2+4s+13}\right) = \frac{3s+1}{s^2+4s+13}$$

$$V_x(s) = \frac{3(s+2)}{(s+2)^2+3^2} - \frac{\frac{5}{3}(3)}{(s+2)^2+3^2}$$

$$v_x(t) = e^{-2t}\left[3\cos(3t) - \frac{5}{3}\sin(3t)\right]u(t)$$

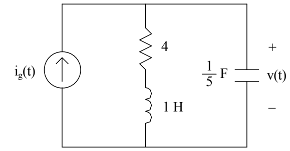
$$v(t) = \frac{v_x}{2} = e^{-2t}\left[1.5\cos(3t) - \frac{5}{6}\sin(3t)\right]u(t)\text{ V}$$

or $v(t) = 1.716e^{-2t}\cos(3t + 29.1°)$ V $= y(t)$

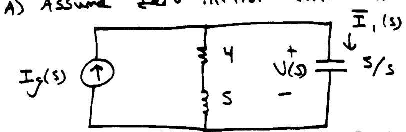
4) A) Determine the transfer function $H(s) = V(s)/I_g(s)$ for the circuit shown below.

B) Determine the impulse response [i.e., find $v(t)$ if $i_g(t) = \delta(t)$ A].

C) Determine the unit step response [i.e., find $v(t)$ if $i_g(t) = u(t)$ A].



A) Assume zero initial conditions



By current division: $I_1(s) = I_g(s)\left[\frac{4+s}{4+s+5/s}\right]$

$$V(s) = \frac{5}{s}\cdot I_1(s) = I_g(s)\left(\frac{5}{s}\right)\left[\frac{4+s}{4+s+5/s}\right]$$

$$H(s) = \frac{V(s)}{I_g(s)} = \frac{5s^2+20s}{s(s^2+4s+5)}, \quad \boxed{H(s) = \frac{5s+20}{s^2+4s+5}}$$

B) $i_g(t) = \delta(t)$ so $I_g(s) = 1$; $V(s) = H(s)\,I_g(s) = H(s)$

$v(t) = h(t) = \mathcal{L}^{-1}\{H(s)\}$

H(s) has complex poles so

$$H(s) = \frac{5s+20}{(s+2)^2+1^2} = \frac{5(s+2)}{(s+2)^2+1^2} + \frac{10(1)}{(s+2)^2+1^2}$$

so $\boxed{v(t) = h(t) = \text{impulse response} = e^{-2t}[5\cos t + 10\sin t]\text{ V}}$

C) $i_g(t) = u(t)$, so $I_g(s) = 1/s$

$$V(s) = H(s)\,I_g(s) = \frac{5s+20}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$A = \left.\frac{5s+20}{s^2+4s+5}\right|_{s=0} = 4$$

$$V(s) = \frac{4}{s} + \frac{Bs+C}{s^2+4s+5} = \frac{4s^2+16s+20+Bs^2+Cs}{s^2+4s+5}$$

$(B+4)s^2 = 0$, $\boxed{B=-4}$

$(C+16)s = 5s$, $\boxed{C=-11}$

$$V(s) = \frac{4}{s} + \frac{-4s-11}{(s+2)^2+(1)^2} = \frac{4}{s} + \frac{-4(s+2)}{(s+2)^2+1^2} + \frac{-3(1)}{(s+2)^2+1^2}$$

so $\boxed{v(t) = \text{unit step response} = \left(4 - e^{-2t}[4\cos t + 3\sin t]\right)u(t)\text{ V}}$