

Test #1 Overview

Material Covered:

Homework #1 - Chapter 12 (Laplace transforms)

Homework #2 - Chapter 13 (Circuit analysis using Laplace transforms)

Sections 12.1-12.9, 13.1-13.5, 13.7 in Electric Circuits, 8th Edition, by Nilsson

Sections 4.1-4.4 in Linear Signals & Systems, 2nd Edition, by Lathi

Exam Materials:

- No calculators allowed
- Handout of Laplace Transform Properties and Common Laplace Transform Pairs provided (attached)
- No other formulas, books, etc allowed on the exam

Laplace Transforms and Inverse Laplace Transforms:

- Finding Laplace transforms:
 - By definition (only for simple functions)
 - No questions on convergence
 - Using known transforms and properties (primary method for finding F(s))
 - Find F(s) if a graph of f(t) is given
- Finding Inverse Laplace Transforms
 - Partial Fractions Expansion (PFE)
 - Use either of two methods to find PFE coefficients (your choice):
 - residue method
 - common denominator method
 - Be sure that order of N(s) < order of D(s) before using PFE – if not use long division
 - Complex roots – can be handled in two ways:
 - quadratic factors – put complex denominator in the form $(s + a)^2 + w^2$ – easiest method without a calculator
 - complex linear roots - simplify as much as possible if not using a calculator
- Trigonometric identities provided if necessary.
- No problems on solving differential equations using Laplace Transforms

Circuit Analysis using Laplace Transforms

- Find initial conditions ($v_C(0)$ and $i_L(0)$) only if not provided
- Draw the Laplace Transformed Circuit (know the models for each component)
- Use basic circuit analysis techniques – your choice of method generally
- Partial solution may be specified – such as find V(s) or I(s) – in simplified polynomial form

Transfer functions:

- Finding transfer functions ($H(s) = Y(s)/X(s)$) for a given circuit
- Recall that transfer functions are always defined with zero initial conditions (so no sources in the models for L and C)
- Finding an output using a given transfer function – $Y(s) = H(s)X(s)$ so $y(t) = \mathcal{L}^{-1}\{H(s)X(s)\}$
- Finding the impulse response for a given circuit or transfer function: impulse response = $h(t) = \mathcal{L}^{-1}\{H(s)\}$
- Finding the unit step response for a given circuit or transfer function: unit step response = $y(t) = \mathcal{L}^{-1}\{H(s)/s\}$

LAPLACE TRANSFORM PROPERTIES

1. **Linearity:** $\mathcal{L}\{af(t)\} = aF(s)$
2. **Superposition:** $\mathcal{L}\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$
3. **Modulation:** $\mathcal{L}\{e^{-at}f(t)\} = F(s + a)$
4. **Time-Shifting:** $\mathcal{L}\{f(t - \tau)u(t - \tau)\} = e^{-s\tau}F(s)$
5. **Scaling:** $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$
6. **Real Differentiation:** $\mathcal{L}\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0)$
7. **Real Integration:** $\mathcal{L}\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$
8. **Complex Differentiation:** $\mathcal{L}\{tf(t)\} = -\frac{d}{ds}F(s)$
9. **Complex Integration:** $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)ds$
10. **Convolution:** $\mathcal{L}\{f(t) * g(t)\} = F(s) \cdot G(s)$

COMMON LAPLACE TRANSFORM PAIRS

f(t)	F(s)
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$e^{-at}u(t)$	$\frac{1}{s + a}$
$te^{-at}u(t)$	$\frac{1}{(s + a)^2}$
$\cos(\omega t)u(t)$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t)u(t)$	$\frac{\omega}{s^2 + \omega^2}$