

Test #2 Overview

Chapters covered: 3 & 4 from Statics, 8th Edition, by Beer & Johnston

Breakdown: Chapter 3 material: $\approx 50\%$
Chapter 4 material: $\approx 50\%$

Format: Similar to class examples, homework problems, and textbook problems

Hints for success: Work more problems for preparation.
Include Free Body Diagrams (FBDs) in your solution.
Show clear diagrams and all work on the test.

Chapter 3 (Rigid Bodies – Equivalent Systems of Forces) - Major Topics

Moment = torque = twisting action (about a point)

A moment is a vector quantity.

In 2D it is expressed by a magnitude and a direction: CW(-) or CCW(+)

Note: the direction is determined by inspection

In 3D it is expressed using rectangular components

Determining moments in 2D - 4 methods:

- 1) $M = (F_{\perp})D$ {the perpendicular component of the force multiplied by the distance}
- 2) $M = F(D_{\perp})$ {the force multiplied by the perpendicular component of the distance}
- 3) Using rectangular components for both F and D
- 4) Using a cross product

Determining moments in 3D - Use a cross product: $\bar{M}_A = \bar{r} \times \bar{F} = M_x \mathbf{i} + M_y \mathbf{j} + M_z \mathbf{k}$

Note the \bar{r} is the position vector from point A to any point along the line of action of F.

Dot Products: $\bar{P} \cdot \bar{Q} = PQ \cos \theta$ (scalar result)

Applications:

- 1) Determining the angle between two vectors

$$\bar{A} \cdot \bar{B} = |\bar{A}| \cdot |\bar{B}| \cdot \cos(\theta)$$

$$\cos(\theta) = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}| \cdot |\bar{B}|} = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

- 2) Determining the projection of a vector along a line

$$P_{OL} = P \cos \theta = \text{projection of } P \text{ along } OL$$

$$\bar{P} \cdot \bar{Q} = PQ \cos \theta$$

$$\frac{\bar{P} \cdot \bar{Q}}{Q} = P \cos \theta = P_{OL}$$

Moment of a force about a line

M_x = moment about the x-axis, M_y = moment about the y-axis, M_z = moment about the z-axis

M_{AB} = moment about the line AB

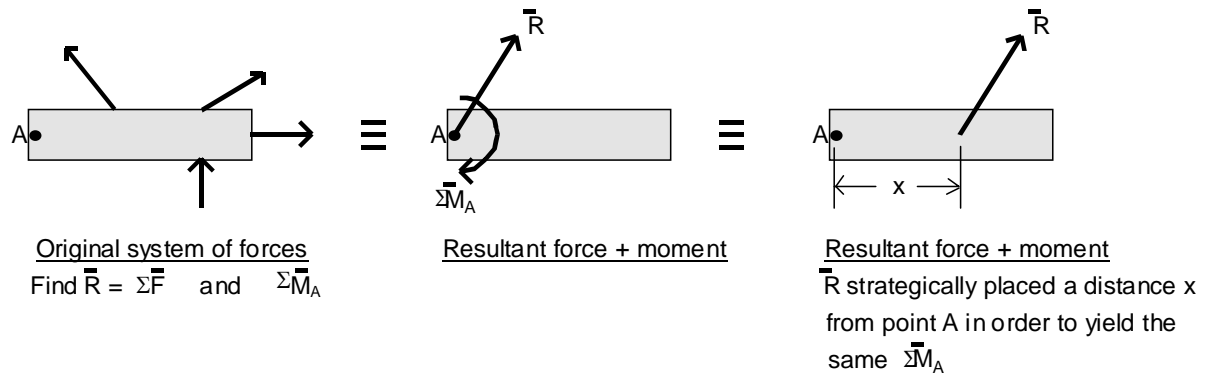
$$|\mathbf{M}_{AB}| = \bar{\lambda}_{AB} \cdot \bar{M}_A = \bar{\lambda}_{AB} \cdot [\bar{r} \times \bar{F}]$$

Couples - A couple is a moment resulting from two forces that are equal in magnitude, opposite in direction, and have different lines of action. A couple is a free vector and is independent of any point, so it may be placed anywhere on an object.

Equivalent Systems - A given system of forces and moments may be represented in two alternate ways:

- 1) As a resultant force and a couple at some point A, where

$$\mathbf{R} = \sum \mathbf{F} \quad (\text{for the original system}) \quad \text{and} \quad M_A = \sum M_A \quad (\text{for the original system})$$
- 2) As a single resultant force strategically placed to produce the same M_A as in the original system.



Chapter 4 (Equilibrium of Rigid Bodies) - Major Topics

Free Body Diagram (FBD) - One of the most important concepts in this course
 Include a FBD with every solution.

Steps to forming a FBD:

- 1) Identify the object to be isolated
- 2) Sketch the object to be isolated with appropriate angles
- 3) Draw vectors representing all external forces acting on the isolated object (including gravitational forces)

2D Equilibrium:

FBD is essential. Refer to Fig. 4.1 for reactions at 2D supports (provided on tests).

2D Equilibrium equations: (3 equations total in most cases)

- most common set of equations: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_A = 0$ (for any point A)
- other possible sets:
 - 1) $\sum F_x = 0$, $\sum M_A = 0$, $\sum M_B = 0$ (A and B not on a vertical line)
 - 2) $\sum F_y = 0$, $\sum M_A = 0$, $\sum M_B = 0$ (A and B not on a horizontal line)
 - 3) $\sum M_A = 0$, $\sum M_B = 0$, $\sum M_C = 0$ (A, B, and C not on any line)

3D Equilibrium:

FBD is essential. Refer to Fig. 4.10 for reactions at 3D supports (provided on tests).

Use cross products in determining moments.

3D Equilibrium equations: (6 equations total in most cases)

$$\sum \bar{\mathbf{F}} = 0 \quad \text{on the free body yields: } \sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$$

$$\sum \bar{\mathbf{M}}_A = \sum (\bar{\mathbf{r}} \times \bar{\mathbf{F}}) = 0 \quad \text{at any point A yields: } \sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$$

Two-force members: Forces act along the axis of the member (the line connecting its endpoints).
 Forces at supports represented by single unknown.

Three-force members: Forces must be either concurrent or parallel.
 Forces at supports represented by both x and y components.