On Line Mathematics Module

All about Fractions

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Defining Fractions

A ***fraction*** is a way of representing division of a 'whole' into 'parts'. It has the form

$\frac{\text{Numerator}}{\text{Denominator}}$	the red line is called a fraction or division bar

Where the Numerator is the number of parts selected ***and*** the Denominator is the number of total parts involved. So in plain words, a fraction is a division problem.

Here are some examples of fractions:

- Example1 $\frac{2}{5}$ In this example 2 is the numerator which is part of the whole which is 5 .Think of a pizza made of 5 slices, and you ate 2 slices. So you ate $\frac{2}{5}$ of the pizza.
- Example2 $\frac{1}{6}$ In this example 1 is the numerator which is part of the whole which is 6 .Think of drinking 1 can of soda out of a 6 can pack. So you drank $\frac{1}{6}$ of six pack. What if you drank 3 cans? This would be $\frac{3}{6}$ or $\frac{1}{2}$ of the six pack. So, saying $\frac{3}{6}$ is the same as $\frac{1}{2}$. We will show you that with the proper rules of simplifications.

Simplifying Fractions

Before we start simplifying fractions, we need to recall and discuss the concept of:

1. ***Multiples***
2. ***Divisibility***
3. ***Factors***.

And why is that important?

Before we discuss simplifying fractions, we need to know the concept of factors

and factoring. This is why we will start with multiples, and divisibility.
Consider the fraction $\frac{12}{36}$. This can be simplified to $\frac{1}{3}$ We shall work this later !

Multiples

When we multiply a natural number n by other natural numbers, we obtain the multiples of t

Note: The Natural numbers are the counting numbers á$1, 2, 3, 4, 5, ..$â

Examples

Consider the natural number 3.

¾ $1 \times 3 = 3$
¾ $2 \times 3 = 6$
¾ $3 \times 3 = 9$ and so on.
¾ Therefore, the numbers $3, 6, 9, 12, ...$ are some of the multiples of 3.
¾ So we can find multiples of three by continued counting by 3.
¾ Now try for yourself to find the multiples of 5.

Answers $1 \times 5 = 5$

$2 \times 5 = 10$

$3 \times 5 = 15$

and so on..

So the multiples are $5, 10, 15, 20, ...$ and keep on adding 5.

Examples Find the Multiples of 7, 4, and ?6 :

Answers

¾ The multiples of 7 are $7, 14, 21, 28, ...$
¾ The multiples of 4 are $4, 8, 12, 16, ...$
¾ The multiples of ?6 are ?6, ?12, ?18, ?24, ... and keep adding ?6

Divisibility

Now that we know what multiples are, we can look the at the divisibility concept as follows:

b is divisible by a, if b is a multiple of a where a & b are natural numbers.

Examples

Consider the ***natural*** number:

1. $b = 6$, we say 6 is divisible by 2 because from above we know that 6 is a multiple of 2.

2. $b = 15$, we say 15 is divisible by 5 because from above we know that 15 is a multiple of 5.

Now try it for yourself to find if the number 36 is divisible by 9.

Here is the **S**olution :

Since 9 is a multiple of 36

i.e.

$1 \times 9 = 9$

$2 \times 9 = 18$

:

:

$4 \times 9 = 36$

We say 36 is divisible by 9.

Factors & Prime factorization

Factors We make use of the previous information and we say,

c is a factor of b, if b is divisible by c. Here b & c are numbers.

A factorization of b means we can express b as a product natural numbers.

Examples

Consider the ***natural*** number $b = 12$.

1. $2 \times 6 = 12$ Therefore 2 and 6 are both factors of 12.

2. $3 \times 4 = 12$ Therefore 3 and 4 are both factors of 12.

Similarly, $1 \times 12 = 12$ and therefore the positive factors of 12 are $1, 2, 3, 4, 6, 12$

Now try it for yourself to find if the factors of the number 18.

Prime Factors and Prime Factorization

¾ What is a prime number?

It is a natural number that has only 2 unique factors, 1 and itself. 1 is not prime by definition. Examples : 3 is a prime because $3 = 1 \times 3$ also 5 is prime because $5 = 1 \times 5$

However 6 is not prime. Why?

1. $6 = 1 \times 6$
2. $6 = 2 \times 3$ so the factorization is not unique and we call 6 a composite number.
3. If not prime, a ***natural*** number called composite.

Simplify

In order to simplify a fraction, both numerator and denominator need to written in factored form, and preferably in a prime factored form.

Consider the following fraction $\frac{15}{25}$. Is it in the simplest form? Is it reduced completely? Do the numerator and denominator have factors that are common to both?
All the above questions leads to the same diagnosis. We have to prime factor both, top and bottom of the fraction bar. Observe the factors (prime), cancel any that are in both.

Examples $\frac{15}{25} = \frac{3\times5}{5\times5} = \frac{3}{5} \times \frac{5}{5} = \frac{3}{5}$ we cancelled $\frac{5}{5}$ since they are the same ($\frac{5}{5} = 1$Þ

$\frac{18}{42} = \frac{2\times3\times3}{2\times3\times7} = \frac{3}{7}$ we cancelled $\frac{3\times3}{3\times7} = 1$

Now back to our original problem that we promised to do earlier in the lesson. Consider the fraction $\frac{12}{36}$. This can be simplified to $\frac{1}{3}$

$$\frac{12}{36} = \frac{2\times2\times3}{2\times2\times3\times3} = \frac{2\times2\times3}{2\times2\times3} \times \frac{1}{3} = 1 \times \frac{1}{3} = \frac{1}{3}$$

Good Luck with you studying!